3.4 Further Trigonometry

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.4 Further Trigonometry
Difficulty	Very Hard

Time allowed: 80

Score: /64

Percentage: /100

Question 1a

In each of the following, θ is an angle measured in radians such that $0 < \theta < \frac{\pi}{2}$.

(a) Given that $\sin\theta = p$, write down expressions for $\sin(\pi - \theta)$, $\cos(\pi - \theta)$ and $\tan(\pi - \theta)$.

[3 marks]

Question 1b

(b) Given that $\cos\theta = q$, write down expressions for $\sin(\pi + \theta)$, $\cos(\pi + \theta)$ and $\tan(\pi + \theta)$.

[3 marks]

Question 1c

(c) Given that $\tan\theta = r$, write down expressions for $\sin(2\pi - \theta)$, $\cos(2\pi - \theta)$ and $\tan(2\pi - \theta)$.

[4 marks]

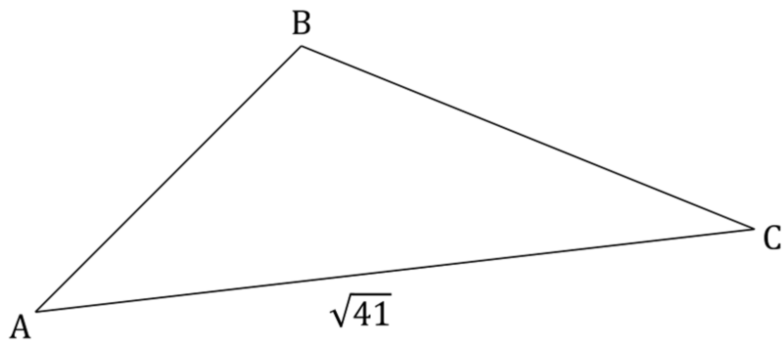
Question 2

Given that $\tan\theta = -\frac{4}{3}$, find the possible values of $\sin\theta$ and the corresponding values of $\cos\theta$.

[5 marks]

Question 3

The following diagram shows triangle ABC, with $AC = \sqrt{41}$.



Given that $\tan A\hat{B}C = -\dfrac{12}{5}$ and that the ratio of the length of side AB to the length of side BC is $10 : 13$, find the exact area of triangle ABC.

[8 marks]

Question 4

A sector of a circle, OPQ, is such that the angle at its centre, O, is $\frac{11\pi}{14}$ radians.

Given that the area of sector OPQ in cm^2 is equal to the length of the arc PQ in mm, find

(i) the area of sector OPQ, and

(ii) the length of arc PQ

giving your answers correct to 3 significant figures.

[6 marks]

Question 5

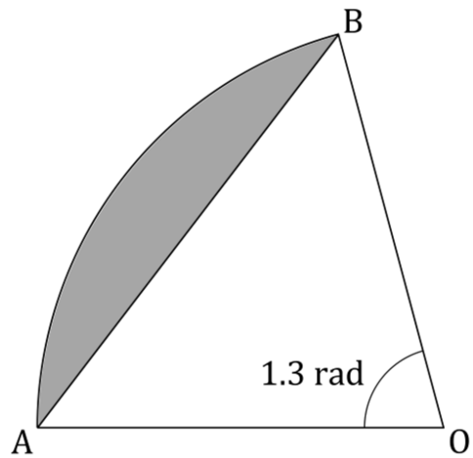
The lengths of two sides in a right-angled triangle are x and y, with $\sqrt{2}x < y$.

Find the possible values of $\sin\theta$, and the corresponding values of $\cos\theta$ and $\tan\theta$, where θ is the smallest angle in the triangle. All your answers should be given in terms of x and y.

[9 marks]

Question 6

The diagram below shows the sector of a circle OAB, with centre O. The angle at the centre of the sector, $A\hat{O}B$, is 1.3 radians. The shaded region in the diagram is the segment bounded by the arc AB and the chord AB.

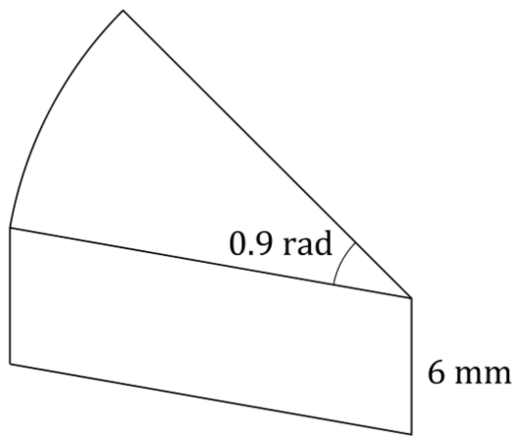


Given that the difference between the perimeter of sector OAB and the perimeter of triangle OAB is 1.05 cm, find the area of the shaded region. Give your answer correct to 3 significant figures.

[8 marks]

Question 7

A games design company produces a popular game called 'Nugatory Enterprise'. Each game set includes plastic game pieces which are in the form of a right prism with a cross-section that is the sector of a circle, as shown in the diagram below. The angle at the centre of the sector is 0.9 radians, and the height of the game piece is 6 mm.



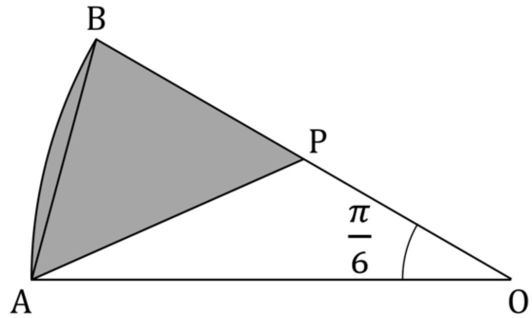
The game pieces are hollow, with a top (which is a cross-section of the prism) and three sides, but no bottom.

Given that the external surface area of the game piece is 4.26 cm^2, work out the interior volume of the game piece giving your answer correct to 3 significant figures. You may ignore the thickness of the top and sides in your calculations.

[8 marks]

Question 8

The diagram below shows the sector of a circle OAB with centre O. The angle at the centre of the sector, $A\hat{O}B$, is $\frac{\pi}{6}$ radians. Point P is a point on line segment [OB] such that $OP = k \times OB$, where k is a constant with $0 < k < 1$. The shaded region in the diagram is the combination of triangle ABP with the region enclosed by the arc AB and the chord AB.



(a) If the area of triangle OAP is denoted by a and the area of the shaded region is denoted by b, show that

$$k = \left(\frac{a}{a+b}\right)\frac{\pi}{3}$$

[6 marks]

Question 9

Solve the equation

$$\sin x \tan x = \frac{1}{2\cos x}$$

in the interval $-180° \leq x \leq 90°$.

[4 marks]