3.11 Vector Planes

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.11 Vector Planes
Difficulty	Hard

Time allowed: 110

Score: /88

Percentage: /100

Question 1a

The points A(2, 1, 0), B(-1, 4, 1) and C(1, 0, 3) lie on a plane Π.

a)
Find an equation for Π in the form $ax + by + cz = d$ where $a,\ b,\ c,\ d \in \mathbb{Z}$.

[7 marks]

Question 1b

(b)
Determine whether the point D(-2, 2, 5) lies on Π.

[2 marks]

Question 2a

The plane Π has equation $r \cdot \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} = 8$.

The line L has equation $r = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$.

The plane Π and the line L intersect at the point X.

(a)
Find the coordinates of X.

[3 marks]

Question 2b

(b)
Find the acute angle, in degrees, between the line L and the plane Π.

[4 marks]

Question 2c

The point P(1,-3, 1) lies on the line L.

(c)
Find the exact value of PX.

[2 marks]

Question 2d

(d)
Hence find the shortest distance between the point P and the plane Π.

[2 marks]

Question 3

Find the acute angle, in radians, between the two planes Π_1 and Π_2 which can be defined by the equations:

$$\Pi_1 : 5x - 2y + z = 19$$

$$\Pi_2 : \mathrm{r} \cdot \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} = 20.$$

[5 marks]

Question 4a

The line *L* given by the Cartesian equation $\frac{x-1}{2}=\frac{3-y}{3}=z+2$ lies on the plane Π. The point P(4, 0, -3) also lies on Π.

(a)

Show that the vectors $\begin{pmatrix}2\\-3\\1\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ are parallel to Π.

[3 marks]

Question 4b

(b)

Hence find the Cartesian equation of Π .

[4 marks]

Question 5a

Consider the plane Π defined by the Cartesian equation $2x - 5y + 3z = 19$ and the line L_1 defined by the vector equation

$$r = \begin{pmatrix} 7 \\ -4 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}.$$

(a)
Show that the line L_1 is parallel to the plane Π but does not lie in the plane.

[3 marks]

Question 5b

The line L_2 is perpendicular to the plane Π and passes through the point P(7, -4, 9) .

(b)
Find a vector equation of the line L_2.

[2 marks]

Question 5c

(c)
Find the coordinates of the point where the line L_2 and the plane Π intersect.

[3 marks]

Question 5d

(d)

Hence find the shortest distance between the line L_1 and the plane Π.

[2 marks]

Question 6a

Consider the two planes defined by the Cartesian equations:

$$\Pi_1 : 2x + y + 2z = 8$$

$$\Pi_2 : 3x - y - 2z = 7.$$

The line L is the intersection of the planes Π_1 and Π_2.

(a)

Show that the line L is parallel to the vector $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$.

[3 marks]

Question 6b

The point P$(a, 0, b)$ lies on both planes.

(b)
(i)
Find the values of a and b.
(ii)
Hence write down a vector equation of the line L.

[3 marks]

Question 6c

A third plane Π_3 has the Cartesian equation $2x - 3y + z = 14$.

(c)
Use algebra to show that the three planes intersect at a unique point Q and find the coordinates of Q.

[4 marks]

Question 7a

Consider the three planes with Cartesian equations:

$$\Pi_1 : 2x + 3y + kz = 11$$

$$\Pi_2 : 3x + y - z = -8$$

$$\Pi_3 : x - 5y + 2z = 15$$

where k is a real constant.

(a)
In the case when the three planes do not intersect at a unique point, find the value of k and state the geometrical relationship between the three planes.

[6 marks]

Question 7b

(b)
In the case when $k = 0$ find the coordinates of the point of intersection between the three planes.

[2 marks]

Question 8a

Two parallel planes are defined by the equations:

$$\Pi_1 : \mathrm{r} \cdot \begin{pmatrix} 7 \\ -4 \\ a \end{pmatrix} = 113,\ a \in \mathbb{R},$$

$$\Pi_2 : \mathrm{r} = \begin{pmatrix} 11 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} b \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -2 \\ -3 \end{pmatrix},\ b \in \mathbb{R}.$$

(a)
Show that $a = 19$ and find the value of b.

[3 marks]

Question 8b

(b)
Write down a vector equation of the line L that is perpendicular to both planes and goes through the point P(11, -3, 5).

[2 marks]

Question 8c

(c)
Find the coordinates of the point where the line L intersects the plane Π_1.

[3 marks]

Question 8d

(d)
Hence find the shortest distance between the two planes Π_1 and Π_2.

[2 marks]

Question 9a

The plane Π has the vector equation $\mathrm{r} = \begin{pmatrix} 6 \\ -19 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 8 \\ -1 \end{pmatrix}$.

(a)
Find a vector that is perpendicular to the plane Π.

[2 marks]

Question 9b

(b)
Q is the point on the plane Π that is closest to the point P(4, 0, -3). Find the coordinates of the point Q.

[4 marks]

Question 9c

(c)
Hence find the reflection of the point P in the plane Π.

[3 marks]

Question 10a

Two planes are defined by the Cartesian equations:

$$\Pi_1 : x - 2y + 3z = 11$$

$$\Pi_2 : 3x + 4y - z = 3.$$

(a)
Find the acute angle, in radians, between Π_1 and Π_2.

[3 marks]

Question 10b

A third plane Π_3 is defined by the equation $5x + ky + z = 13$ where $k \in \mathbb{R}$.

(b)
The plane Π_3 is perpendicular to the plane Π_1. Find the value of k.

[2 marks]

Question 10c

(c)
(i)
Describe the geometrical configuration of the three planes.
(ii)
Find the acute angle, in radians, between Π_2 and Π_3.

[4 marks]