

4.4 Probability Distributions

Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.4 Probability Distributions
Difficulty	Medium

Time allowed: 70
Score: /51
Percentage: /100

Question 1a

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

[1 mark]

Question 1b

For each coin the probability of getting heads is $\frac{2}{3}$. A random variable, X , is defined as the number of heads when the three coins are tossed.

(b) Complete the following probability distribution table for X :

x	0	1	2	3
$P(X = x)$				

[3 marks]

Question 1c

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for X as a piecewise function in the form

$$P(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2 marks]

Question 1d

(d) Represent the probability distribution for X as a bar chart.

[2 marks]

Question 2

The random variable X has the probability function

$$P(X = x) = \begin{cases} \frac{x}{3k} & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that $k = 5$.

[2 marks]

Question 3a

The random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k .

[2 marks]

Question 3b

(b) Find $P(X > 3)$.

[2 marks]

Question 3c

(c) State, with a reason, whether or not X is a discrete random variable.

[1 mark]

Question 4a

The random variable X has the probability function

$$P(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k .

[2 marks]

Question 4b

(b) Construct a table giving the probability distribution of X .

[2 marks]

Question 4c

(c) Find $P(0 \leq X < 3)$.

[1 mark]

Question 5

A discrete random variable X has the probability distribution shown in the following table:

x	0	1	2	3	4
$P(X = x)$	$\frac{5}{24}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{8}$

Find:

- (i) $P(X < 4)$
- (ii) $P(X > 1)$
- (iii) $P(2 < X \leq 4)$
- (iv) $P(0 < X < 4)$

[6 marks]

Question 6a

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of p .

[1 mark]

Question 6b

Leonardo is playing a game with his biased spinner. The score for the game, X , is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

[3 marks]

Question 6c

(c) Complete the following cumulative probability function table for X :

Score x	0	1	2	3	5
$P(X \leq x)$	$\frac{6}{20}$				1

[2 marks]

Question 6d

(d) Find the probability that X is

- (i) no more than 1
- (ii) at least 3.

[2 marks]

Question 7a

A discrete random variable X has the following probability distribution:

x	-3	-1	0	1	3
$P(X = x)$	0.11	k^2	0.1	$2k$	0.1

where k is a positive constant.

(a) Show that $k^2 + 2k - 0.69 = 0$.

[2 marks]

Question 7b

(b) Hence find the value of k .

[1 mark]

Question 7c

(c) Find $E(X)$.

[3 marks]

Question 8

A spinner is spun on a circle that is divided up into five sections, A, B, C, D and E.

The probability of the spinner landing on each section is given by the following table:

Region	A	B	C	D	E
Probability	0.55	0.15	0.15	0.1	0.05

A person who rotates the spinner scores points depending on which section the spinner lands on. These points are shown below.

Region	A	B	C	D	E
Points	-5	2	3	10	k

Given that the game is fair, find the value of k .

[4 marks]

Question 9a

A discrete random variable X has the following probability distribution:

x	0	1	2	3	4
$P(X = x)$	0.1	0.05	a	b	0.1

The value of $E(X) = 2.3$.

(a) Show that a and b must satisfy the following two simultaneous equations:

$$\begin{aligned}a + b &= 0.75 \\2a + 3b &= 1.85\end{aligned}$$

[3 marks]

Question 9b

(b) Hence find the value of a and the value of b .

[2 marks]

Question 9c

(c) Find $P(1 \leq X < 4)$.

[2 marks]