1.7 Permutations & Combinations

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.7 Permutations & Combinations
Difficulty	Hard

Time allowed: 90

Score: /71

Percentage: /100

Question 1a

A scientist is randomly writing out permutations of the 5 letters and the 2 numbers in the acronym COVID 19.

(a)
Find the number of different arrangements of the seven characters that could be created if
(i)
the number 19 appears in the arrangement with the digits 1 and 9 in that order and no letters in between the two digits
(ii)
the five letters are all together

[3 marks]

Question 1b

(b)
If only the five letters are used, find the number of possible arrangements if (reading from left to right) the letter D must be in a position after the letter C.

[2 marks]

Question 1c

(c)
The scientist arranges the five letters and two numbers so that they make up the seven vertices of a regular heptagon, and then uses straight lines to join each vertex of the heptagon to each other vertex. Use counting principles to explain why there are exactly 21 lines.

[2 marks]

Question 2a

Consider two parallel lines A and B. Line A has 7 distinct points marked on it and line B has 4 distinct points marked on it. The diagram below shows one possible example of this:

Line A   X   X   X   X   X   X   X

Line B   X   X   X   X
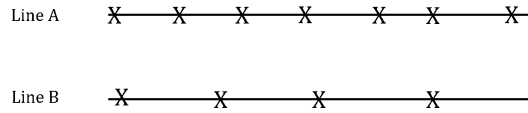
New lines are now drawn between the two parallel lines, so that each marked point on line A is joined to each marked point on line B by exactly one new line.

(a)
Determine the maximum number of points of intersection between the newly constructed lines.

[5 marks]

Question 2b

(b)
Explain why the answer to part (a) is a maximum.

[1 mark]

Question 3

Consider the letters of the word ALGORITHM. Find the number of permutations of three letters that can be chosen if

(i)
no vowels (A, I or O) are in the selection

(ii)
there must be at least one consonant (L, G, R, T, H, M)
(iii)
exactly one vowel is chosen.

[6 marks]

Question 4a

Consider the nine letters in the word MAGNITUDE.

(a)
Find the number of ways that the nine letters may be arranged if
(i)
there are no restrictions
(ii)
the four vowels (A, I, U, E) must all be together

(iii)
the arrangement starts with the letter M and ends with the letter E.

[5 marks]

Question 4b

(b)

Find the probability that if the nine letters in the word MAGNITUDE are arranged randomly, none of the vowels will be next to each other.

[3 marks]

Question 5a

A mixed relay team must consist of four competitors, two of whom must be male and two of whom must be female. There are nine men and six women trying out for a place on a new team.

(a)

During the try-outs the fifteen candidates are split into three groups of four and one group of three. Find the number of ways this could be done if the candidates are divided randomly.

[3 marks]

Question 5b

(b)
Two of the candidates are brother and sister and have agreed that they will only be in the final relay team if they are both successful. Find how many ways the final relay team can be chosen if the brother and sister are either both in or both out.

[3 marks]

Question 6a

An examination paper consists of four questions in section A and eight questions in section B. Candidates must answer five questions from the paper in any order.

(a)
Find the number of ways a candidate can choose their questions if
(i)
there are no restrictions
(ii)
the candidate must choose at least three of their questions from section A.

[3 marks]

Question 6b

(b)
Candidates are now told that if they choose question 1 from section A they cannot choose any other question from section A. However if they do not choose question 1 from section A then they must choose at least two questions from section A and answer question 1 from section B. Find the number of ways a candidate could choose their questions under these restrictions.

[4 marks]

Question 7a

Dylan is preparing a playlist for his friend's birthday party. Dylan chooses 5 different afrobeats songs, 3 different blues song, 3 different country songs and 8 different drum and bass songs.

(a)
Find the number of different orders Dylan can play the songs in if
(i)
there must be a country song at the beginning, the exact middle and the end of the playlist
(ii)
the country songs are all played first, followed by all the blues songs, then all the afrobeats songs, and finally all the drum and bass songs.

[4 marks]

Question 7b

(b)

Whilst at the party a friend likes Dylan's music and decides to use it to make a playlist of her own. The friend only has space for twelve songs on her computer. Find the number of ways Dylan's friend can choose and arrange twelve out of the nineteen songs if she decide to have two blues songs first, followed by alternating afrobeats and drum and bass songs

[4 marks]

Question 8a

In his classroom Mr Roland has seven different books about GeoGebra, five different books about football, and three different Mathematics textbooks.

(a)

Find how many ways Mr Roland could organise the books on his bookshelf if

(i)

there are no restrictions

(ii)

the five books about football must all be separated by at least one other book

(iii)

the first and last books are Mathematics textbooks and the rest are in any order.

Give your answers in the form $a \times 10^k$ .where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[5 marks]

Question 8b

(b)

Mr Roland's young son selects three of the books at random to sit on whilst doing his homework. Find the number of possible different selections from the fifteen books that there are if

(i)

he chooses one of each type of book

(ii)

he chooses two football books and one other book.

[3 marks]

Question 9a

Consider ten cards numbered 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0.

(a)

The ten cards are placed randomly in a line. Find how many ways this can be done if

(i)

no multiples of 3 are next to each other

(ii)

the multiples of 3 are not all together

(iii)

the number formed by the ten cards is greater than 5 000 000 000.

[6 marks]

Question 9b

(b)
Four of the ten cards are chosen at random and placed in a line to make a 4–digit code. Find the number of possible codes that can be made if the code is required to be an even number.

[2 marks]

Question 10a

Helen is researching three different types of tree as part of her IB Biology Internal Assessment. She has collected samples from four different acacia trees, two different banyan trees and three different cedar trees. She needs to choose six of the tree samples to include in her final write up.

(a)
Find the number of different selections of the tree samples that could be made if she must have at least one cedar tree sample but cannot include more than three samples of any single type.

[4 marks]

Question 10b

Helen chooses three acacia samples, two banyan samples and one cedar sample to include in her final write up.

(b)
Find the number of different orders in which she can arrange these samples in a row if the two banyan samples must not be next to each other.

[3 marks]