2.8 Inequalities

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.8 Inequalities
Difficulty	Very Hard

Time allowed: 60

Score: /47

Percentage: /100

Question 1

Consider the functions defined by $f(x) = x^2 - 6ax + b + 10$ and $g(x) = ax + 2b + 3$, where $a, b \in Z^+$. Given that $f(x) \leq g(x)$ only for $2 \leq x \leq 5$, find the values of a and b.

[4 marks]

Question 2a

The function defined by $f(x) = x^4 - 12x^3 + 46x^2 - 60x + 25$ can be factorised into the form $f(x) = (x - a)^2(x - b)^2$, where a and b are positive integers such that $a < b$.

(a)
Find the values of a and b.

[3 marks]

Question 2b

(b)

Determine the set of values of that satisfy

(i)

$f(x) \geq 0$,

(ii)

$f(-x) \geq 0$,

(iii)

$-f(x) < 0$.

[3 marks]

Question 2c

(c)

Determine the smallest positive value k such that the solution to the inequality $f(x) \leq k$ is a single interval.

[2 marks]

Question 3

The function f is such that

$$f(x) \geq 0 \text{ for } x \leq 3 \text{ and for } 4 \leq x \leq 5,$$

$$f(x) \leq 0 \text{ for } 3 \leq x \leq 4 \text{ and for } x \geq 5.$$

Find a polynomial, of the lowest degree possible, that satisfies the condition $f(0) = 5$.

[5 marks]

Question 4a

(a)
Sketch the graph of $y = f(x)$ where

$$f(x) = \frac{(x+2)(x-4)(x-6)}{(x-1)(x-5)}$$

Label any intersections with the coordinate axes and state the equations of any vertical asymptotes.

[3 marks]

Question 4b

(b)
Find the values of x that satisfy
(i)
$f(x) \geq 0.$
(ii)
$f(|x|) \geq 0.$

[5 marks]

Question 5

The region *R* is defined by the three straight lines given by the inequalities

$$y \geq 1,$$

$$y \leq 2x + 8,$$

$$x + y \leq 10.$$

The function f is defined by $f(x) = 2 + \frac{1}{x-1}$. Find the largest domain of f such that the graph of f lies within the region *R*.

Give answers as exact values where appropriate.

[6 marks]

Question 6a

(a)
Consider the graphs with equations

$$y = \frac{(x+4)(x-1)}{x-1} \text{ and } y = 6 - x.$$

Explain why the two graphs do **not** intersect.

[1 mark]

Question 6b

(b)
Consider the graphs with equations

$$y = \frac{(x-6)(x-1)^2}{x-1} \text{ and } y = (8-x)(x-1).$$

(i)
Find the coordinates of any points of intersections between the two graphs.

(ii)
Hence, or otherwise, solve the inequality

$$\frac{(x-6)(x-1)^2}{x-1} \leq (8-x)(x-1).$$

[3 marks]

Question 7a

Consider the functions defined by $f(x) = \sqrt{(9-x^2)}$, $g(x) = 3 - \sqrt{(9-x^2)}$ and $h(x) = \frac{x+3}{2}$. All three functions have the domain $-3 \leq x \leq 3$.

(a)
On the same diagram, sketch the graphs of f, g and h.

[3 marks]

Question 7b

(b)

Find the set of values of x which satisfy the inequality $f(x) > g(x) > h(x)$.

[3 marks]

Question 8

Find the exact values for x such that

$$\frac{x}{(x+2)(x-3)} \geq x$$

[6 marks]