5.1 Differentiation

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.1 Differentiation
Difficulty	Very Hard

Time allowed: 120

Score: /92

Percentage: /100

Question 1a

A curve is given by the equation

$$y = \frac{1}{6}x^3 - \frac{3}{8}x^2 - \frac{3}{2}x + 4$$

(a) Determine the coordinates of the points on the curve where the gradient is 2. You must show all your working, and give your answers as exact fractions.

[6 marks]

Question 1b

(b) Find the range of values for x for which the curve is increasing.

[3 marks]

Question 2a

An engineer is designing a right cone that is to be produced on a 3D printer. The cone has a base radius of r cm and a height of h cm, and while the radius may vary freely the height must always be 7 cm more than the radius.

(a) Write down, in terms of r only, the formula for the volume of the cone.

[2 marks]

Question 2b

(b) Find the exact value of the radius at the point where the instantaneous rate of change of the volume with respect to the radius is $\frac{5\pi}{3}\ \text{cm}^3/\text{cm}$.

[5 marks]

Question 3a

A curve has the equation

$$f(x) = 2x^3 + \frac{3}{x} - 4$$

Points A and B are the two points on the curve where the gradient is equal to 3, and the x-coordinate of A is less than zero.

(a) Find the coordinates of points A and B.

[3 marks]

Question 3b

(b) Find the equations of

(i) the tangent to the curve at point A

(ii) the normal to the curve at point B.

[5 marks]

Question 3c

Point C is the point of intersection of the two lines found in part (b).

(c) Find the coordinates of point C. Give your answers as exact fractions.

[3 marks]

Question 4

A curve has equation $f(x) = ax^2 + bx + c$.

The gradient of the tangent to the curve at the point $(-3, d)$ is 25.

The gradient of the tangent to the curve at the point $(2, -1)$ is -5.

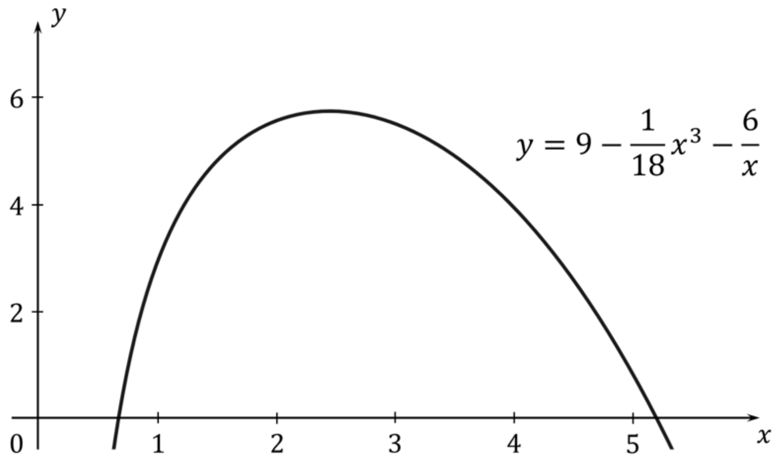
Find the values of a, b, c and d.

[7 marks]

Question 5a

The diagram below shows a part of the graph of the function $y = f(x)$, where

$$f(x) = 9 - \frac{1}{18}x^3 - \frac{6}{x}, \qquad x > 0$$



(a) Calculate the average rate of change of $f(x)$ between $x = 3$ and

(i) $x = 4$

(ii) $x = 3.5$

(iii) $x = 3.25$

[4 marks]

Question 5b

(b) Explain what would happen to the values of the average rates of change in part (b) if you continued to calculate them, moving the second x value closer and closer to 3 each time.

[3 marks]

Question 6a

Let f be a function defined by $f(x) = -6x^3 + 7x^2 + 3x$ for all $x \in \mathbb{R}$.

The curve $y = f(x)$ intercepts the x-axis at points $\mathrm{A}(a, 0)$, $\mathrm{B}(b, 0)$ and $\mathrm{C}(c, 0)$, where $a < b < c$.

(a) Find the values of a, b and c.

[3 marks]

Question 6b

The curve $y = f(x)$ has a local minimum at point D.

(b) Show that the x-coordinate of point D is equal to $\frac{7-\sqrt{113}}{18}$. Be sure to justify that the point corresponding to that x-coordinate is indeed a local minimum, and that it is the only local minimum.

[6 marks]

Question 6c

The curve $y = f(x)$ has a point of inflection at point E.

(c) Find the gradient of the normal to the curve at point E.

[3 marks]

Question 7a

A function f is defined by

$$f(x) = \frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x - \frac{38}{15}$$

for all real numbers x.

(a) (i) Show that the curve $y = f(x)$ has a stationary point when $x = 1$ and determine the corresponding y-coordinate.

(ii) Find the x-coordinates of any other stationary points on the curve.

[6 marks]

Question 7b

(b) Determine the ranges of values of x for which $f(x)$ is

(i) increasing

(ii) decreasing.

[4 marks]

Question 7c

(c) Given that the value of the function when $x = 1$ is greater than the value of the function at any other stationary point, sketch the curve of $y = f(x)$. Be sure to show clearly the x-coordinates of any minimum and maximum points, as well as the coordinates of the point where the curve crosses the y-axis.

[3 marks]

Question 8a

A function f is defined for all for $x \neq 0$. The derivative of f is given by

$$f'(x) = 48x^2 + \frac{1}{x^3} - 22$$

The graph of f is concave up when $x > n$, where n is the least possible number that makes that inequality true.

(a) Find the value of n.

[6 marks]

Question 8b

(b) Show that the curve $y = f(x)$ has only one point of inflection, and find the gradient of the normal line to the curve at that point.

[5 marks]

Question 9a

A curve has equation $y = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$ and $a \neq 0$.

(a) (i) Show that the curve will only have stationary points if a, b and c satisfy the inequality $b^2 \geq 3ac$.

(ii) In the case where the inequality in part (a)(i) is satisfied, determine the x-coordinate(s) of the stationary point(s), giving your answer as simply as possible in terms of a, b and c.

[7 marks]

Question 9b

(b) Show that the curve will always have exactly one point of inflection, and determine its x-coordinate in terms of a and b.

[4 marks]

Question 9c

(c) In the case where the point of inflection is also a stationary point, show that the curve will have no other stationary points.

[3 marks]

Question 9d

(d) In the case where the curve has two distinct stationary points, show that the x-coordinate of the point of inflection will lie halfway between the x-coordinates of the two stationary points.

[1 mark]