5.3 Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.3 Integration
Difficulty	Hard

Time allowed: 80

Score: /62

Percentage: /100

Question 1a

A function f is defined by the equation $f(x) = -3x + 35$.

(a) Sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 10$.

[2 marks]

Question 1b

(b) Use your sketch from part (a), along with relevant area formulae, to work out the value of the integral

$$\int_1^9 (-3x + 35)\, \mathrm{d}x$$

[3 marks]

Question 2

The derivative of the function f is given by

$$f'(x) = \frac{9}{2}x^2 + 7x - 2$$

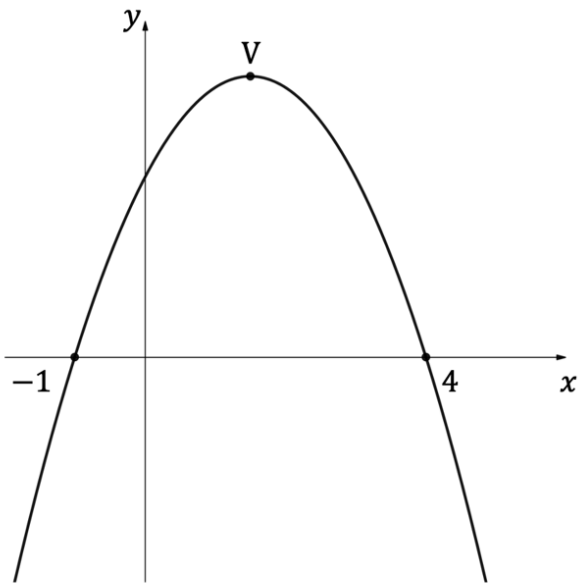
and the curve $y = f(x)$ passes through the point $\left(-3, -\frac{11}{2}\right)$.

Find an expression for f.

[6 marks]

Question 3a

A curve $y = f(x)$ has the gradient function $f'(x) = 4ax + 6$, where $a \in \mathbb{R}$ is a constant. The diagram below shows part of the curve, with the x and y intercepts labelled and where V represents the vertex of the curve.



(a) Find

(i) the value of a

(ii) the equation of the curve $y = f(x)$

(iii) the coordinates of V.

[5 marks]

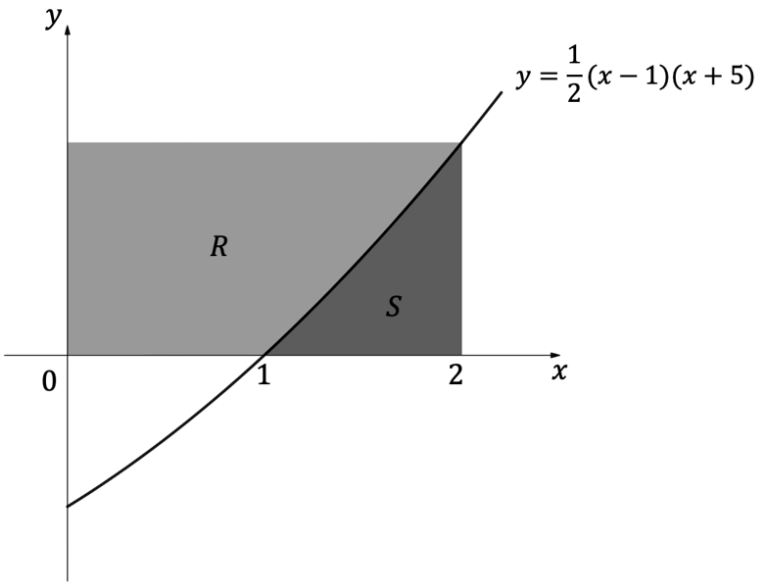
Question 3b

(b) Find the area between the curve and the x-axis.

[3 marks]

Question 4a

A section of the curve with equation $y = \frac{1}{2}(x-1)(x+5)$ is shown below:



The shaded region S in the diagram is bounded by the curve, the x-axis and the line $x = 2$.

(a) (i) Write down an integral for the area of the shaded region S.

(ii) Find the area of S. Give your answer as a fraction.

[3 marks]

Question 4b

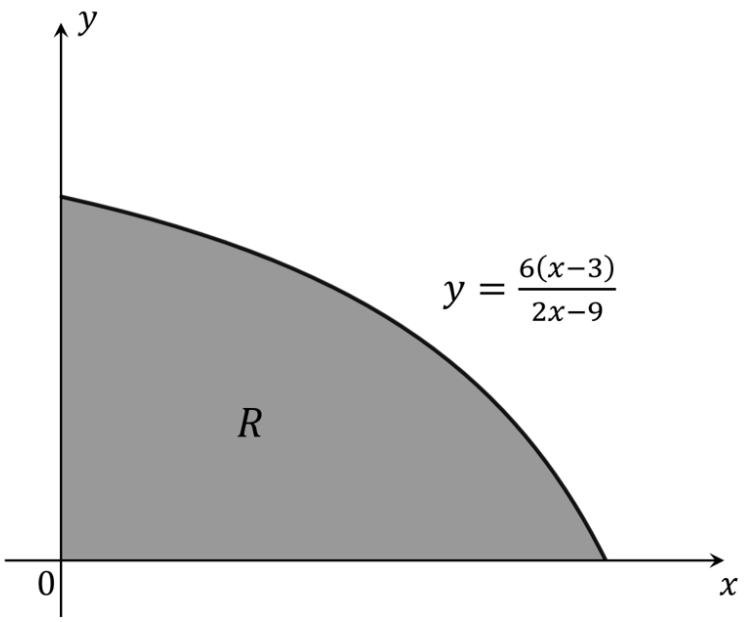
The shaded region R in the diagram is bounded on three sides by the curve, the x-axis and the y-axis. The boundary on the fourth side is a straight line parallel to the x-axis, and that line, the curve and the line $x = 2$ all intersect at a single point.

(b) Find the area of region R. Give your answer as a fraction.

[3 marks]

Question 5a

A company is designing a plastic piece for a new game. The piece is to be in the form of a prism, with a cross-sectional area as indicated by the shaded region R in the following diagram:



Region R is bounded, as shown, by the positive x- and y-axes and the curve with equation $y = \frac{6(x-3)}{2x-9}$. All units are in centimetres.

(a) Using technology, or otherwise, find the coordinates of the points of intersection of the curve with the x- and y-axes.

[2 marks]

Question 5b

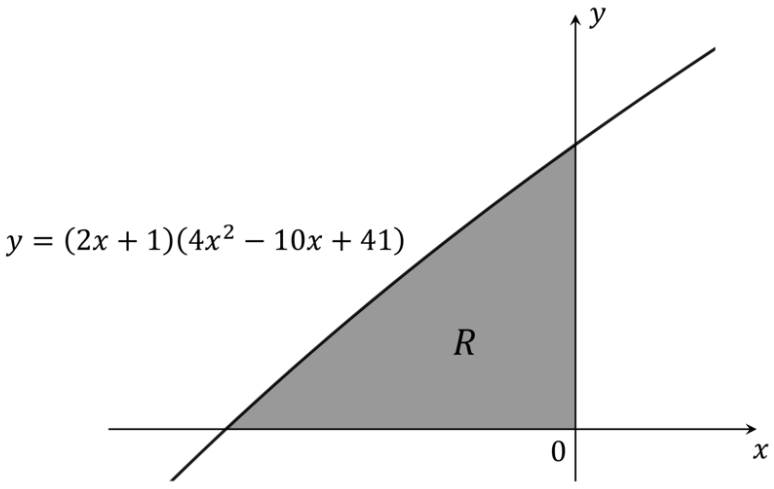
The volume of the puzzle piece is to be $30\ \text{cm}^3$.

(b) Find the length of the puzzle piece, giving your answer correct to 3 significant figures.

[4 marks]

Question 6a

The following diagram shows part of the graph of $f(x) = (2x + 1)(4x^2 - 10x + 41),\ x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.



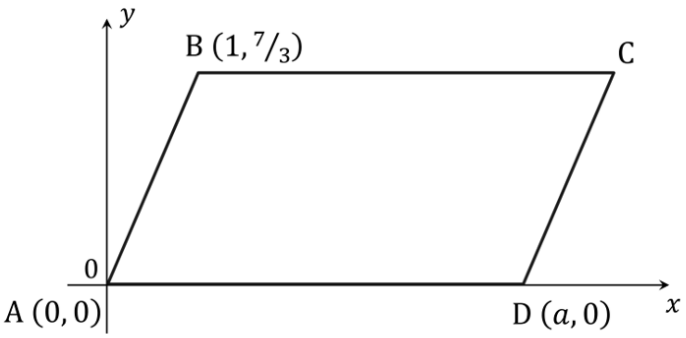
(a) (i) Write down an integral for the area of region R.

(ii) Find the area of region R.

[3 marks]

Question 6b

ABCD is a parallelogram with vertices A(0,0), B $\left(1, \frac{7}{3}\right)$, C and D$(a, 0)$, as shown in the diagram below. The area of ABCD is equal to the area of region R above.

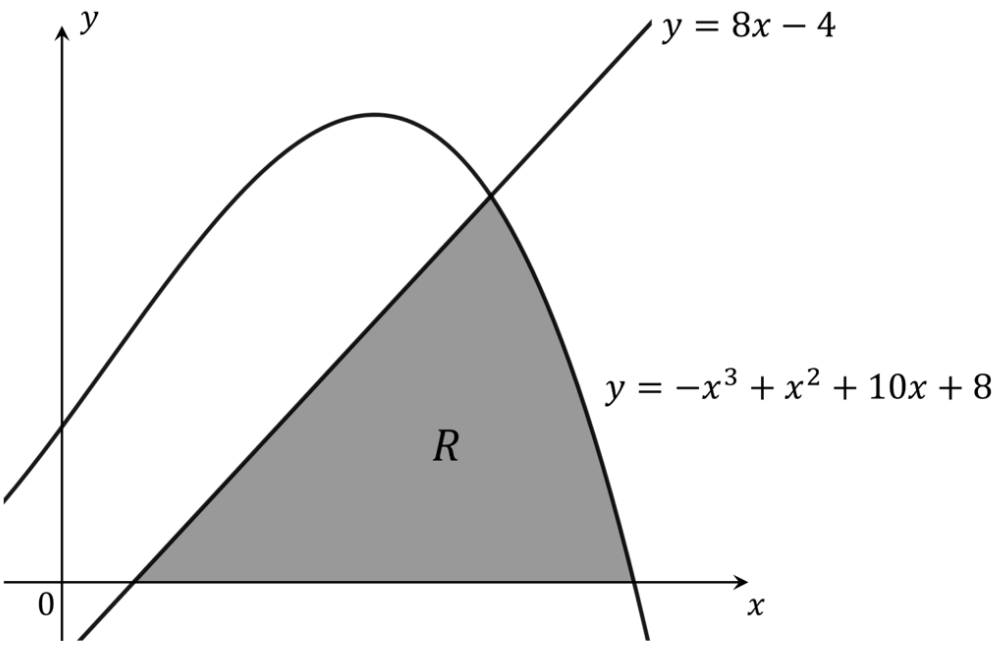


(b) By first finding the value of a, the x-coordinate of point D, determine the coordinates of point C. The coordinates should be given as exact fractions.

[3 marks]

Question 7a

The shaded region R in the following diagram is bounded by the x-axis, the line $y = 8x - 4$ and the curve $y = -x^3 + x^2 + 10x + 8$.



(a) Using technology, or otherwise, find the coordinates of

(i) the point of intersection between the curve and the line

(ii) the point of intersection between the line and the x-axis

(iii) the point of intersection between the curve and the x-axis that is shown in the diagram.

[3 marks]

Question 7b

(b) Show that the area of region R is equal to exactly $\frac{439}{12}$ units^2. Be sure to show all of your working.

[6 marks]

Question 8a

Consider the function f where $f(x) = x(x^2 - 12) + 16,\ x \in \mathbb{R}$.

The turning points on the graph of f are A and B. The x-coordinates of points A and B are a and b respectively, where $a < b$.

(a) (i) Determine an expression for the derivative of f.

(ii) Hence find the values of a and b, and the coordinates of points A and B.

[4 marks]

Question 8b

Point C is the point on the graph with x-coordinate c, where $c > 0$ and $f(c) = 32$.

(b) (i) Determine the value of c.

(ii) Sketch the graph of f, clearly indicating the locations of points A, B and C, along with all other points where the graph intersects one of the coordinate axes.

[3 marks]

Question 8c

Region R is the region enclosed by the graph of f and the line $y = 32$.

(c) Find the area of region R.

[4 marks]

Question 9a

For a particle P travelling in a straight line, the velocity, v m/s, of the particle at time t seconds is given by the equation

$$v(t) = 2t^2 - 8t + 9, \qquad t \geq 0$$

(a) Sketch the graph of $v(t)$ in the interval $0 \leq t \leq 5$.

[2 marks]

Question 9b

The distance travelled between times t_1 and t_2 by a particle moving in a straight line may be found by finding the area beneath the particle's velocity-time graph between those two times.

(b) Find the distance travelled by the particle P between the times $t = 1$ and $t = 4.5$.

[3 marks]