3.10 Graph Theory

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.10 Graph Theory
Difficulty	Hard

Time allowed: 80

Score: /65

Percentage: /100

Question 1a

Let G be a graph with 5 vertices. The adjacency matrix of G is shown below.

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	0
C	0	1	0	1	0
D	1	0	0	0	1
E	1	0	0	0	1

a)
Explain what the fact that the adjacency matrix is not symmetrical about the leading diagonal means with respect to the associated graph.

[1 mark]

Question 1b

b)
Draw the graph of G.

[3 marks]

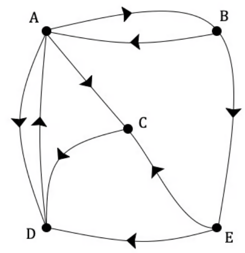
Question 1c

c)
Determine whether G is Eulerian, semi-Eulerian or neither. Justify your answer.

[2 marks]

Question 2a

Consider the graph G shown below.



a)

Write down the adjacency matrix for the graph G.

[3 marks]

Question 2b

b)

Find the total number of walks of length 7 that start at vertex A and finish at vertex D.

[2 marks]

Question 2c

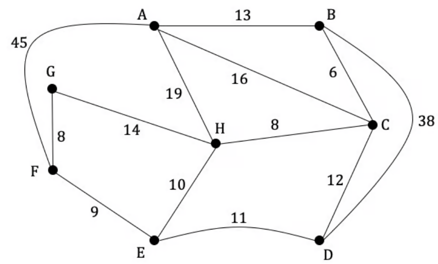
c)

Find the total number of walks of length 7 that start at vertex D and finish at vertex A. Explain any difference found between this result and the value found in part (b).

[2 marks]

Question 3a

Let G be the graph shown below. In the graph the vertices represent different clinics in a hospital and the edges represent the possible locations for pipelines to supply oxygen to the clinics. The weighting of the edges represents the cost, in hundreds of US dollars, of installing each pipe.



a)
Show that G is a Hamiltonian graph.

[1 mark]

Question 3b

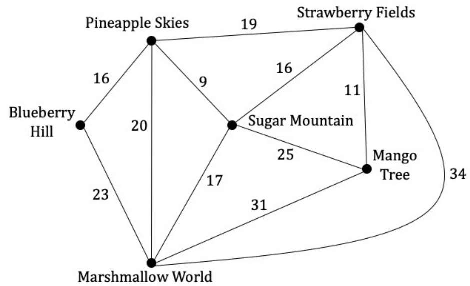
b)
Using Kruskal's algorithm, find the minimum cost of a network that will connect each clinic to the oxygen supply.

[4 marks]

Question 4a

The musical elves who live in the magical land of Fooditooti want to build a network of rainbow bridges to connect the different cities in the land, and they wish to do so in such a way as to minimise the total length of the bridges that they need to build.

The graph G below shows a schematic map of Fooditooti, along with its 6 cities: Pineapple Skies, Strawberry Fields, Marshmallow World, Sugar Mountain, Blueberry Hill and Mango Tree. The weighting of the edges between the cities represents the length in kilometres of the rainbow bridge that would be required to connect the two cities.



a)
Starting from Pineapple Skies and using Prim's algorithm, find a network of bridges of minimum total length that will span the whole land.

[5 marks]

Question 4b

b)
Given that rainbow bricks are sold in bundles, with each bundle costing $15 and containing enough bricks to build 42 centimetres of rainbow bridge, find the total cost of constructing the network from part (a).

[2 marks]

Question 5a

On Monday Dinesh has 5 lessons to attend: English, Biology, French, Maths and Art. His lessons all take place in the different departments in his school, which are connected by a number of corridors. Some of the corridors are narrow so students are only allowed to walk in one direction along them.

From the Biology department, there are corridors that lead to the English, French and Maths departments.

The Maths department is also connected to the Art department and the French department, but students are not allowed to access the Maths department from the French department.

The English department has corridors connecting it to the French department and the Art department, although students cannot walk back through the corridor from the Art department to the English department.

a)
Construct a graph showing how the classrooms are connected.

[3 marks]

Question 5b

b)
Write down a possible path starting from and finishing at the Maths department that Dinesh could take to visit each of the departments without using the same corridor more than once.

[1 mark]

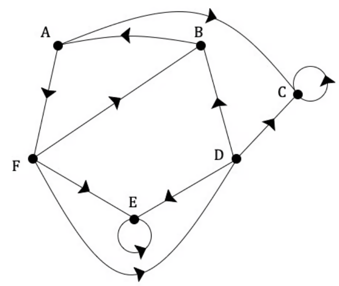
Question 5c

c)
Given that the timetable requires Dinesh to go to his lessons in the order initially stated, write down the route that he should take to minimise the number of corridors that he must walk down more than once.

[2 marks]

Question 6a

Consider the graph G below.



a)
For a random walk through graph G, explain why the long-term probabilities will be dependent on the starting position.

[1 mark]

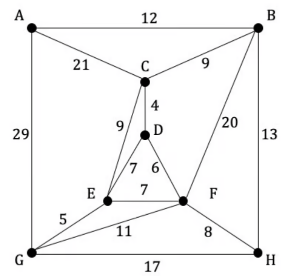
Question 6b

b)
Find the probability of a random walk finishing at vertex C given that the starting position is at vertex F.

[6 marks]

Question 7a

The graph *P*, shown below, represents the layout of a sculpture park with each vertex representing the position of a sculpture and the edges representing the pathways between the sculptures. The weighting of each edge indicates the length of the corresponding pathway in metres.



A treasure hunt has been set up for visitors with different tokens to collect hidden along the pathways.

a)
State with a reason whether *P* contains an Eulerian circuit.

[1 mark]

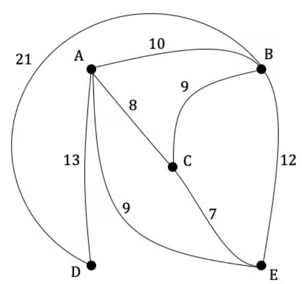
Question 7b

b)
Given that a visitor must start and finish at the same sculpture, find the minimum distance that a visitor must travel in order to walk along each edge at least once in order to find all the tokens. State clearly which edges must be repeated.

[7 marks]

Question 8a

In the graph below each vertex represents a place of interest in a museum, with the weighted edges representing the pathways connecting the places of interest and their lengths in metres. Vertices A and E represent the entrance hall and the souvenir shop, respectively.



Other than possibly starting and finishing at the same location, Elijah wishes to visit each place of interest once and only once during his visit.

a)
Using the graph above, write down a possible route for Elijah
(i)
starting at the entrance hall and finishing at the souvenir shop
(ii)
starting and finishing at the souvenir shop.

[2 marks]

Question 8b

b)
Explain why there must be fewer than 24 possible routes meeting Elijah's criteria that start and finish from the same vertex in the graph.

[1 mark]

Question 8c

c)

By inspection, find the shortest distance that Elijah will need to travel given that he must start and finish at the entrance hall. State the route that he should take.

[4 marks]

Question 9a

In order to distribute food for the animals in the wildlife park that she runs, Jessamy needs to visit each of the enclosures A, B, C, D and E. She starts at enclosure as this is where the feed is stored, then visits each enclosure once and returns to to put back the feeding equipment. As this is a task that must be repeated twice a day, Jessamy wishes to minimise the time that it takes to visit all of the different enclosures.

The weighted graph G, with weights representing the time in minutes taken to move between the five enclosures, can be represented by the following table:

	A	B	C	D	E
A	-	7	5	8	8
B	7	-	11	12	14
C	5	11	-	3	12
D	8	12	3	-	9
E	8	14	12	9	-

a)
Draw the weighted graph G.

[2 marks]

Question 9b

Let *T* be the shortest possible time for Jessamy's journey between the different enclosures.

b)
Starting at enclosure C, use the nearest neighbour algorithm to find an upper bound for *T*. Indicate clearly the order in which the edges are selected.

[5 marks]

Question 9c

c)
By first removing vertex *E*, use the deleted vertex algorithm to find a lower bound for *T*.

[4 marks]

Question 9d

d)
Hence write down an inequality that is satisfied by *T*.

[1 mark]