

# 4.10 Poisson Distribution

## Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.10 Poisson Distribution
Difficulty	Very Hard

**Time allowed:** 90  
**Score:** /72  
**Percentage:** /100

### Question 1a

Determine, with a reason, whether or not the variable  $X$  in each of the following scenarios can be modelled by a Poisson distribution,  $Po(m)$ . In each case where a Poisson distribution could be appropriate, specify any assumptions that would need to be made and identify the value for  $m$  that could be used.

a)

Harry is walking along a beach in a straight line and finds on average 5 shells every 100 metres.  $X$  is the distance Harry walks after finding a shell until he finds another shell.

[2 marks]

### Question 1b

b)

Each week Hermione donates £5 to a children's charity.  $X$  is the amount of money that Hermione donates to the charity within a 10-week period.

[2 marks]

### Question 1c

c)

On average, Ron receives 7.2 emails every 45 minutes and Ginny receives 4.1 emails every 30 minutes.  $X$  is the combined number of emails that Ron and Ginny receive within a 60-minute period.

[2 marks]

### Question 1d

d)

Neville works an 8-hour shift at a call centre. Neville receives calls from customers at an average rate of 12.4 calls per hour.  $X$  is the number of calls that Neville receives during the last hour of his shift.

**[2 marks]****Question 2a**

Phoebe has a faulty smoke detector which rings at an average rate of 5 times a day.

a)

Write down two conditions for a Poisson distribution to be a suitable model for the number of times that Phoebe's smoke detector rings.

**[2 marks]****Question 2b**

a)

(i)

Find the probability that Phoebe's smoke detector rings exactly 3 times everyday over a four-day period.

(ii) Find the probability that Phoebe's smoke detector rings exactly 12 times in a four-day period.

(iii) Explain why the answers to part (b)(i) and part (b)(ii) are different.

**[4 marks]**

**Question 2c**

c)  
Given that Phoebe's smoke detector rang at least once in a 6-hour period, find the probability that Phoebe's smoke detector rang no more than 4 times during that period.

**[4 marks]****Question 3a**

The number of mistakes made by a student, Priya, in a 20-minute revision period is modelled as a Poisson distribution with a mean of 1.2. The number of mistakes made by a different student, Qays, in a 30-minute revision period is modelled as a Poisson distribution with a mean of 2.2.

a)  
Find the probability that Priya makes exactly 2 mistakes and Qays makes exactly 1 mistake within a one-hour revision period. State any assumptions that are needed.

**[7 marks]**

### Question 3b

b)

The number of mistakes made by Priya and Qays in a one-hour revision period are added together. Given that they make exactly 9 mistakes in total in a one-hour revision period, find the probability that Priya made exactly 5 mistakes in that same revision period.

[3 marks]

### Question 3c

c)

Given that Priya makes exactly 5 mistakes in a one-hour revision period, find the probability that Priya and Qays made exactly 9 mistakes in total in that same revision period.

[2 marks]

### Question 3d

c)

In a week before a test, Priya has ten 30-minute revision periods. Estimate the number of these revision periods during which Priya will make a mistake.

[2 marks]

### Question 4a

Whilst writing an essay, Gamu notices that she makes spelling mistakes at a rate of 7 for every 150 words. Gamu models the number of spelling mistakes she makes using a Poisson distribution.

a)

Find the maximum number of words Gamu can write before the probability of her making a spelling mistake exceeds 0.75.

**[3 marks]**

### Question 4b

Gamu is asked to write three short essays by her lecturer. She writes one containing 100 words, one containing 200 words and one containing 250 words. An essay is returned by Gamu's lecturer if more than 1% of its words contain spelling mistakes.

b)

Find the probability that

(i)

all three short essays are returned.

(ii)

exactly one of the three short essays is returned.

**[6 marks]**

### Question 5a

Caroline is observing cars as they stop at a stop sign outside of her shop. She models the number of cars that stop outside her shop in a given time period using a Poisson distribution. Caroline calculates that cars stop outside her shop at an average rate of 25.4 cars per hour.

Caroline observes how many cars stop at the stop sign outside her shop each hour within a four-hour period.

a)

Find the probability that:

(i)

fewer than 100 cars stop outside the shop within the four hours.

(ii)

at least 20 cars stop outside the shop within each hour of the four-hour period.

(iii)

more than 90 cars stop outside her shop within the four hours given that exactly 35 cars stop outside her shop within the first two hours.

**[6 marks]**

**Question 5b**

The number of people that walk past Caroline's shop in an hour can be modelled as a Poisson distribution with mean 37.9. Caroline hands out a flyer advertising her shop to each person that walks past and to the driver of each car that stops outside her shop.

- b)  
Caroline starts the day with 300 flyers. Find the probability that Caroline has no flyers left after five hours. State an assumption that is needed.

**[4 marks]****Question 5c**

- c)  
Caroline decides to hand out two flyers to the driver of each car that stops outside her shop and only one to each person that walks past. Explain, with a reason, whether or not a Poisson distribution could be used to model the total number of flyers that Caroline hands out within an hour.

**[2 marks]**



### Question 6a

Whilst playing the video game *Thirsty Turkeys*, a player receives points for various achievements. Each week, Jamie plays the game for one hour each day from Monday to Friday. From historical data, Jamie has found that the number of points he gets in an hour can be modelled by a Poisson distribution with mean 21. The number of points that Jamie gets on any one day is independent of the number of points he got on previous days.

If Jamie gets more than 25 points in a day then he gets a bronze star award. If Jamie gets a bronze star award on at least three out of the five days in a week he gets a silver star award. If Jamie gets a bronze star award on at least three consecutive days in a week then he gets a gold star award.

a)

Find the probability that:

(i)

Jamie gets a silver star award in a given week

(ii)

Jamie gets a gold star award in a given week

(iii)

Jamie gets a gold star award in a given week, given that he gets a silver star award that same week.

**[7 marks]**

### Question 6b

If Jamie gets at least 125 points in a five-day week then his name will appear on the leaders' board.

b)

Given that Jamie's name appears on the leaders' board one week, find the probability that he does not get any bronze star awards that week.

[4 marks]

### Question 7a

In 1898, statistician Ladislaus Bortkiewicz made one of the first applications of the Poisson distribution to a real-life situation. He observed 10 Prussian army units over a 20-year period and recorded how many soldiers died each year in each unit by accidentally being kicked by a horse. His data is shown below:

Number of accidental deaths by horse kick reported in a unit in a year	Frequency
0	109
1	65
2	22
3	3
4	1
5 or more	0

a)

Use the data above to calculate unbiased estimates for the mean and variance of the number of accidental deaths by horse-kick reported in a unit in a year. Explain with a reason whether or not this supports Bortkiewicz's suspicion that the data follows a Poisson distribution.

[3 marks]

### Question 7b

Roger wants to make a fictional cartoon about unicorns accidentally poking their riders with their horns. He decides to allow the number of times per episode that characters accidentally get poked by a unicorn horn to be determined randomly, using a Poisson distribution with the mean from part (a). Roger plans to make three episodes. The Unicorn Protection Agency warn Roger that they will file a complaint if there is more than one instance of a character being poked by a unicorn's horn in any one episode, or if there are more than two such instances in total throughout the three episodes.

b)

Find the probability that the Unicorn Protection Agency will file a complaint.

**[5 marks]**