3.4 Further Trigonometry

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.4 Further Trigonometry
Difficulty	Hard

Time allowed: 90

Score: /67

Percentage: /100

Question 1

Complete the following table. In all cases the values for the angle should be given between 0 and 360° or 0 and 2π radians, as appropriate.

Degrees	Radians	sin	cos	tan
45°				1
		$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	
150°		$\frac{1}{2}$		
		-1		
	$\frac{7\pi}{4}$			

[5 marks]

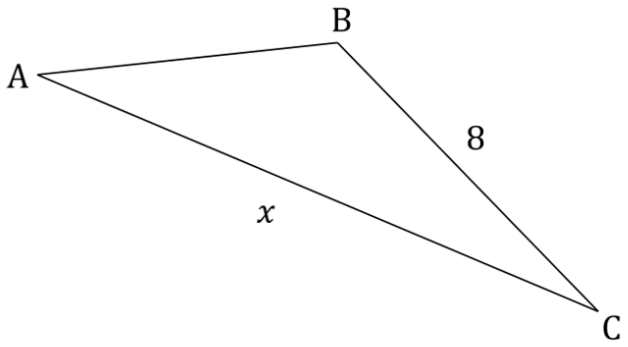
Question 2

Given that $\sin\theta = -\frac{12}{13}$, find the possible values of $\cos\theta$ and the corresponding values of $\tan\theta$.

[4 marks]

Question 3

The following diagram shows triangle ABC, with $\text{BC} = 8$ and $\text{AC} = x$.

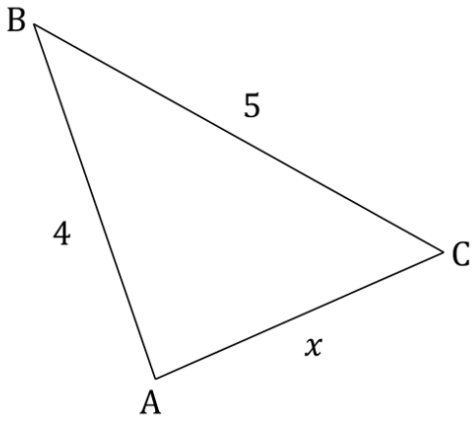


Given that $\tan \hat{\text{ACB}} = \frac{5}{12}$ and that the area of triangle ABC is equal to 20 units^2, find the value of x.

[6 marks]

Question 4a

The following diagram shows triangle ABC, with $\text{AB} = 4$, $\text{BC} = 5$, and $\text{AC} = x$.



(a) Given that $\cos \widehat{\text{ABC}} = \frac{3}{4}$, find the exact area of triangle ABC.

[6 marks]

Question 4b

(b) Find the exact perimeter of triangle ABC.

[4 marks]

Question 5

A sector of a circle, OPQ, is such that the angle at its centre, O, is $\frac{5\pi}{6}$ radians.

The area of sector OPQ in cm^2 is one-fifth of the length of the arc PQ in cm.

(i) Show that the radius of the sector is equal to 0.4 cm, and hence

(ii) find the area of sector OPQ and the length of arc PQ.

Give your answers in part (ii) correct to 3 significant figures.

[6 marks]

Question 6a

Wynken, Blynken and Nod are three mathematics students. While the three are revising trigonometry, Nod sets the following problem for his two companions:

> "ABC is a triangle with $AC = 8.1$ cm, $BC = 9.8$ cm and $A\widehat{B}C = 47°$. To three significant figures, what is the size of the largest angle in the triangle?"

Wynken and Blynken set to work, and several minutes pass. "70.8 degrees," states Wynken confidently. "118 degrees," insists Blynken a moment later.

(a) Demonstrate that Wynken's and Blynken's responses may both be correct answers to the problem Nod has set them.

[6 marks]

Question 6b

(b) Suggest an additional piece of information that Nod could provide, that would allow his problem to have a single unique solution.

[1 mark]

Question 7

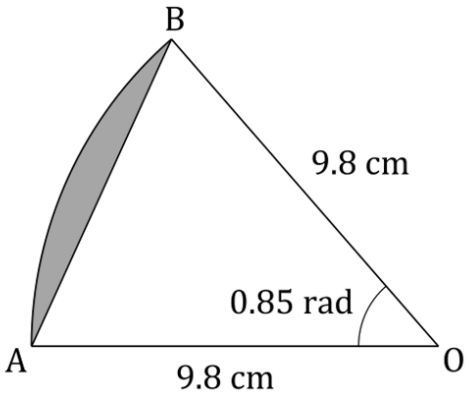
The lengths of two sides in a right-angled triangle are 9 cm and 12 cm.

Find the possible values of $\sin\theta$, and the corresponding values of $\cos\theta$ and $\tan\theta$, where θ is the smallest angle in the triangle. All your answers should be given as exact values.

[7 marks]

Question 8a

The diagram below shows the sector of a circle OAB, with centre O and radius 9.8 cm. The angle at the centre of the sector, $A\widehat{O}B$, is 0.85 radians.



(a) Find the area of the shaded segment, bounded by arc AB and chord AB. Give your answer correct to 3 significant figures.

[5 marks]

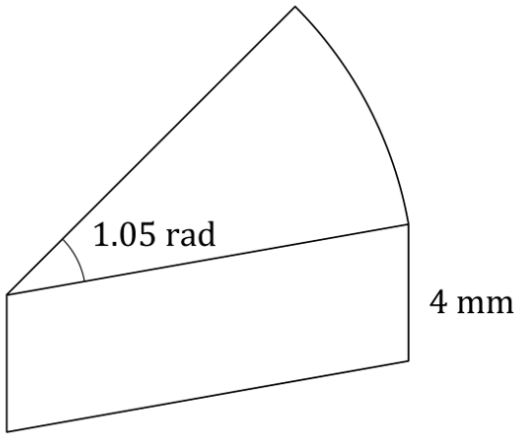
Question 8b

(b) Find the perimeter of sector OAB.

[3 marks]

Question 9

A games design company produces a popular game called 'Inconsequential Endeavour'. Each game set includes solid plastic game pieces which are in the form of a right prism with a cross-section that is the sector of a circle, as shown in the diagram below. The angle at the centre of the sector is 1.05 radians, and the height of the game piece is 4 mm.

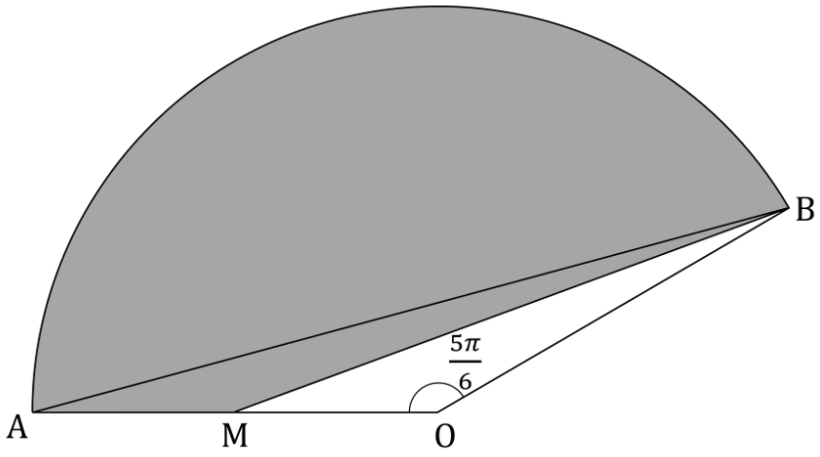


Given that the volume of the game piece is 0.412 cm^3, work out the radius of the sector. Give your answer correct to 3 significant figures.

[5 marks]

Question 10a

The diagram below shows the sector of a circle OAB with centre O. The angle at the centre of the sector, $A\widehat{O}B$, is $\frac{5\pi}{6}$ radians. Point M is the midpoint of line segment OA, and the shaded region is the combination of triangle ABM with the region enclosed by the arc AB and the chord AB.



(a) Show that the ratio of the area of triangle OMB to the area of the shaded region may be expressed as

$$1 : \left(\frac{10\pi}{3} - 1\right)$$

[6 marks]

Question 10b

(b) Given that the area of the shaded region is equal to $30\pi - 9$ units^2, find the exact area of triangle OAB.

[3 marks]