3.4 Voronoi Diagrams

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.4 Voronoi Diagrams
Difficulty	Hard

Time allowed: 70

Score: /51

Percentage: /100

Question 1a

The points $A(4, 3)$, $B(-2, -1)$ and $C(2, -5)$ on the Voronoi diagram below represent the locations of three gold coins in a computer game. Points are scored for each gold coin that is collected by an avatar. Each step that is moved reduces the energy levels of the avatar so it is advised to collect the coin that is closest to you. Each horizontal and vertical unit indicated on the graph represents 1 cm.
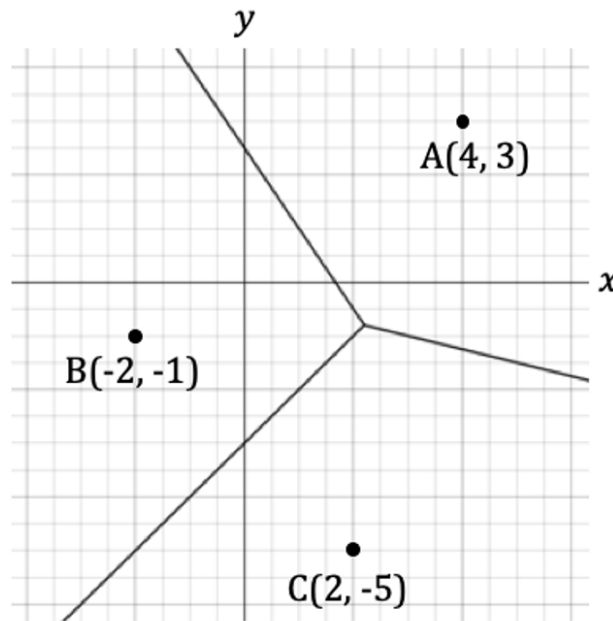
(a) Your avatar is located at $(-1, 3)$.

    (i)   State which gold coin is closest to you.

    (ii)   Calculate the distance from your avatar to the nearest gold coin.

[3 marks]

Question 1b

A new gold coin appears at D$(-2, 4)$.

(b) Redraw the Voronoi diagram on the grid above to incorporate the new gold coin.

[4 marks]

Question 1c

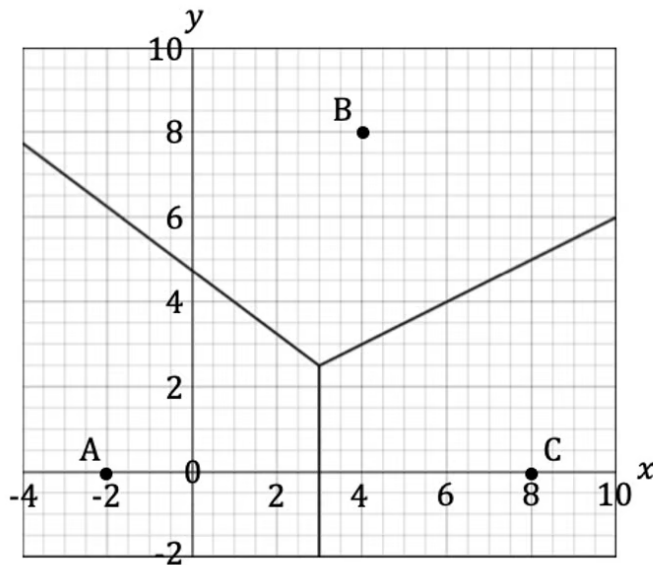(c) Show that the new gold coin is now the closest to your avatar.

[2 marks]

Question 2a

A town is built in a rectangular area bounded by the lines $y = -2, y = 10, x = -4$ and $x = 10$ as shown on the Voronoi diagram below.

Each horizontal and vertical unit on the grid below represents 1 km.

Points A$(-2, 0)$, B$(4, 8)$ and C$(8, 0)$ represent the locations of first aid responders in the town.

(a) Calculate the area for which the first aid responder at C has responsibility.
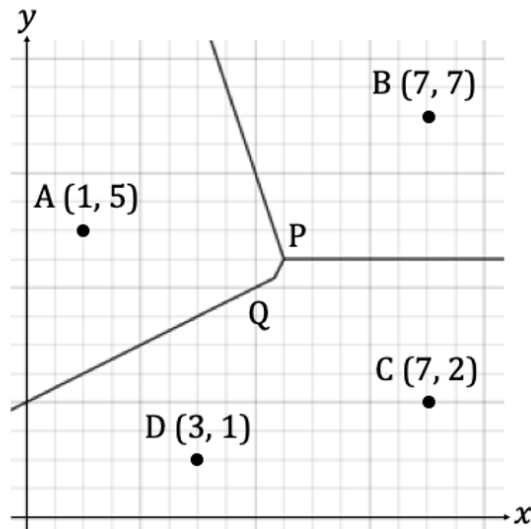
[3 marks]

Question 2b

A new station is due to open at D(8, 2) and as such the Voronoi diagram will need to be adjusted.

(b) (i)   Write down the equation of the perpendicular bisector of [BD]. Give your answer in the form $y = mx + c$.

(ii) Write down the equation of the perpendicular bisector of [CD]. Give your answer in the form $y = mx + c$.

[3 marks]

Question 3a

You have been challenged to stay in a tiger infested jungle overnight. The incomplete Voronoi diagram below shows the known position of four tigers. Each unit on the diagram represents 400 m on the ground.

(a) Find the equation of the line that would complete the Voronoi cell containing the tiger at point D. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[4 marks]

Question 3b

(b) The equation of the perpendicular bisector of [AD] is $x - 2y + 4 = 0$. Determine the coordinates of:

    (i)   Point P.

    (ii) Point Q.

[4 marks]

Question 3c

(c) State which of the two points, P or Q, is the location that is between the tigers but as far as possible from them and calculate the distance that would be between you and the nearest tiger.

[3 marks]

Question 4a

In a square city measuring 15 km by 15 km, there are sixteen post boxes in a grid formation that are spaced at 3 km intervals.

(a) Draw a sketch of a Voronoi diagram showing how the area of the city is served by the different post boxes.

[2 marks]

Question 4b

(b) Calculate the size of the largest and the smallest areas that are each served by a single post box.

[2 marks]

Question 4c

It is decided that the number of post boxes in the city will be reduced and re-arranged. There are now only five post boxes available. One is located at the very centre of the city with the other four located at a distance of 5 km North, South, East and West of the centre.

(c) Draw a sketch of the new Voronoi diagram that would describe this situation.
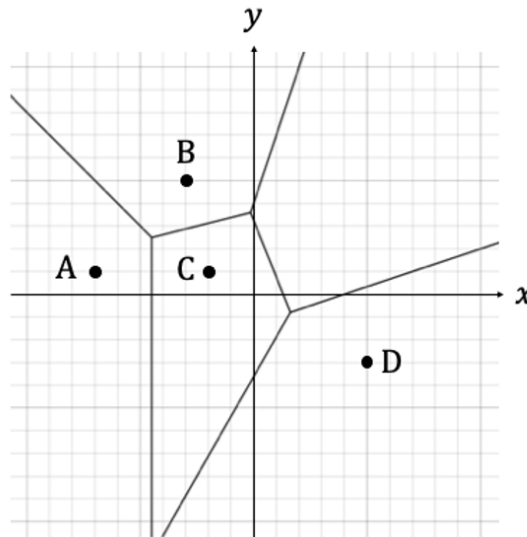
[2 marks]

Question 4d

(d) Calculate the largest area that is now served by a single post box.

[2 marks]

Question 5a

An incomplete Voronoi diagram of a region on the moon's surface is shown below. Each of the five points $A(-7, 1)$, $B(-3, 5)$, $C(-2, 1)$, $D(5, 3)$ and $E(a, b)$ represent the flag of a country that has claimed a portion of the moon. One of the points is missing from the diagram.

(a) Write down the coordinates of the missing flag, E, and complete the diagram.

[2 marks]

Question 5b

Point P has coordinates $P\left(-\dfrac{9}{2}, \dfrac{5}{2}\right)$.

(b) Explain the significance of the location of point P.

[1 mark]

Question 5c

Countries A, B, C and E are upset that country D has obtained the largest area and want D to reposition their flag.

(c) Explain how the territories claimed by C and E will be affected if D moves its flag to $(3, -3)$.

[3 marks]

Question 6a

The diagram below shows how a square area of land has been divided up for the houses of three families at points $A(1, 8)$, $B(7, 8)$ and $C(9, 4)$. Perpendicular bisectors between the houses have been used to form the boundaries of the land surrounding each house. One unit on the diagram corresponds to 10 m in real life.
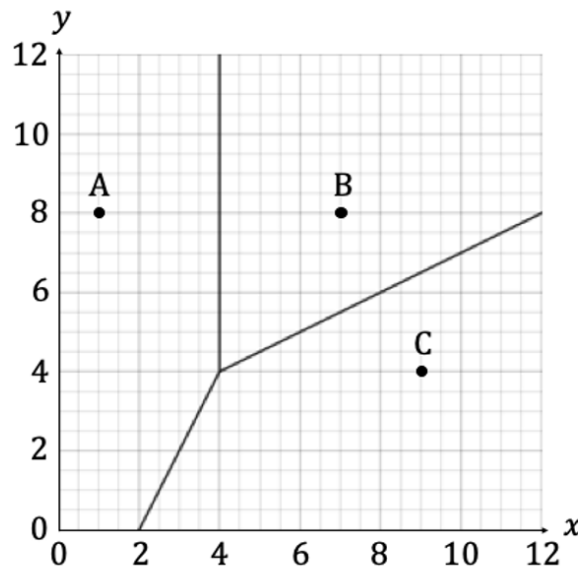
(a) Calculate the area of land that each family own.

[4 marks]

Question 6b

The children from house A and house B fall in love and decide to build a house together in the area between their families.

(b) By redrawing the Voronoi diagram, show that by building their house at D(4, 8), the children will reduce the land surrounding house A more than the land surrounding house B.

[4 marks]

Question 6c

(c) The family at house C complain that with the adjustment of the boundary lines, they lose 3.25% of their land. Show that this statement is true.

[3 marks]