4.4 Probability Distributions

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.4 Probability Distributions
Difficulty	Very Hard

Time allowed: 100

Score: /77

Percentage: /100

Question 1a

Ben plays a game involving a biased eight-sided die.
The faces of the die are labelled $-4,\ -2,\ -1,\ 0,\ 1,\ 3,\ 5,\ 6$.
The score of the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score, x	-4	-2	-1	0	1	3	5	6
$\mathbf{P}(X = x)$	$\frac{1}{6}$	p	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{1}{48}$	q

It is given that $p = 4q$.

(a) Calculate the exact values of p and q.

[2 marks]

Question 1b

Ben plays the game once.

(b) Calculate the expected score.

[3 marks]

Question 1c

Ben plays the game twice and adds the scores together.

(c) Find the probability that Ben has a total score of -3, giving your answer as a fraction in its simplest form.

[3 marks]

Question 2a

A weekly lottery ticket costs \$15, with five different levels of prizes, \$$s$. The grand prize in the first week is \$2000 and it increases by 20% each week that nobody wins it.
The following table shows the probability distribution for S.

Prize, \$$s$	0	2	10	20	100	Grand prize
$\mathbf{P}(S = s)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{24}$	p	$\frac{1}{1000}$

(a) Find the value of p.

[1 mark]

Question 2b

(b) Determine if the lottery is a fair game in the first week.

[2 marks]

Question 2c

(c) Given the grand prize is not won, write an expression in terms of n for the value of the grand prize in the nth week of the lottery.

[2 marks]

Question 2d

The wth week is the first week which a player is expected to make a profit.

(d) Calculate the value of w and the expected profit. Give your answer correct to 2 decimal places.

[4 marks]

Question 3a

A discrete random variable X has the following probability distribution:

x	-5	-3	-1	0	1	3	5
$\mathbf{P}(X = x)$	0.42	$\frac{k^4}{2}$	0.05	0.21	$\left(\frac{3k}{4}\right)^2$	0.09	0.07

(a) Find the value of k.

[4 marks]

Question 3b

(b) Find $\mathrm{E}(X)$.

[3 marks]

Question 4a

A discrete random variable X has the following probability distribution:

x	0	1	2	3	4
$\mathbf{P}(X = x)$	0.04	0.35	a	0.21	b

The value of $\mathrm{E}(X) = 2.28$.

(a) Write down two equations connecting a and b.

[4 marks]

Question 4b

(b) Hence find the value of a and the value of b.

[2 marks]

Question 4c

(c) Find $\mathrm{P}(1 < X \leq 4)$.

[2 marks]

Question 5a

Two fair six-sided dice are rolled. One is a standard die with sides numbered 1 to 6 and the other die has sides numbered 1, 1, 2, 2, 4, 4.

The discrete random variable S is the sum of these two dice when they are rolled.

(a) Complete the following probability distribution table.

s									
$\mathbf{P}(S = s)$									

[5 marks]

Question 5b

(b) Find $\mathrm{E}(S)$.

[3 marks]

Question 6a

Let X be the discrete random variable represented in the probability distribution table below.

x	0	1	2	3	4	5	6	7	8
$\mathbf{P}(X = x)$	0.32	0.22	0.21	$\frac{1}{k}$	$\frac{7}{k^2}$	$\frac{4}{k^2}$	$\frac{2}{k^2}$	$\frac{1}{k^2}$	$\frac{1}{k^2}$

(a) Find the value of k.

[3 marks]

Question 6b

(b) Find the expected value of X.

[3 marks]

Question 7a

The table below represents the number of pets and the corresponding probability of a house having that number of pets.

Number of pets, x	0	1	2	3	4
$\mathbf{P}(X = x)$	0.44	0.21	0.19	p	0.02

(a) Find the value of p.

[1 mark]

Question 7b

(b) Find the expected number of pets in a house.

[2 marks]

Question 7c

There was a recording error and houses with 5 pets were **not counted**. It is found that there are 6 houses with 5 pets. The neighbourhood in total has 406 houses, including these 6 houses.

(c) Complete the following table for the true probability distribution of the number of pets, x.

Number of pets, x	0	1	2	3	4	5
$\mathbf{P}(X = x)$						

[3 marks]

Question 7d

(d) Find the actual expected number of pets.

[2 marks]

Question 7e

(e) Calculate the percentage error between your answer in part (b) and your answer in part (d).

[2 marks]

Question 8a

The table below represents the probability distribution for the number of apple products people have in a city in France.

Number of apple products, x	$\mathbf{P}(X = x)$
0	0.1925
1	0.1815
2	0.2250
3	p
4	0.0895
5	0.0504
6	0.0307
7	0.0104
8	q

It is given that $p = 21q$

(a) Find the value of p and q.

[2 marks]

Question 8b

(b) Find the expected number of apple products a randomly selected person from this city has.

[2 marks]

Question 8c

The city has a population of 412 000. The average amount someone from this city spends on an apple product is €825.

(c) Estimate the revenue apple earned from sales to people in this city. Give your answer to the nearest euro (€).

[2 marks]

Question 9a

A discrete random variable X has the probability distribution shown in the following table:

x	-5	-1	2	6
$\mathbf{P}(X = x)$	$\frac{2}{5}$	$\frac{1}{4}$	p	$4p$

(a) Find the value of p.

[2 marks]

Question 9b

X is sampled twice such that the results of the two experiments are independent of each other and the outcomes of the two experiments are recorded. A new random variable, Y, is defined as the sum of the two outcomes .

(b) Draw a probability distribution table for Y.

[6 marks]

Question 10a

The table below represents the number of strokes Josh takes on a particular round of golf and the corresponding frequencies over a year of playing at the same golf course.

Number of strokes, x	Frequency
70	1
71	7
72	10
75	3
76	2
78	1
80	1

(a) Complete the following probability distribution table for the data above.

Number of strokes, x	$\mathrm{P}(X = x)$
70	
71	
72	
75	
76	
78	
80	

[4 marks]

Question 10b

Par refers to the number of strokes a golfer is expected to need to complete the play on a golf course. The par number of strokes for Josh's golf course is 72.

(b) Determine whether Josh's expected number of strokes is less than or greater than the par number of strokes.

[3 marks]