4.5 Binomial Distributions

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.5 Binomial Distributions
Difficulty	Hard

Time allowed: 100

Score: /83

Percentage: /100

Question 1a

The germination rate of a particular seed is 44%. George sows 25 of these seeds selected at random.

(a) Calculate the expected number of seeds that germinate.

[2 marks]

Question 1b

(b) Find the probability that more than 7 of these seeds will germinate.

[2 marks]

Question 1c

(c) Find the probability that at least 9, but no more than 11 of the seeds germinate.

[3 marks]

Question 2a

At a builders' convention there is competition in which 17 contestants get to choose one out of ten gift boxes. Two of the gift boxes have a chainsaw and the others have a regular saw. After a builder selects a box and claims the gift, the saw is replenished so that the next builder has the same options.

(a) Find the probability that a contestant gets a regular saw.

[1 mark]

Question 2b

(b) Find the probability that exactly five of the 17 contestants win a chainsaw.

[2 marks]

Question 2c

(c) Find the probability that at least three and no more than nine contestants win a chainsaw.

[3 marks]

Question 3a

A biased dice is rolled 24 times. The probability that the dice lands on a prime number is $\frac{3}{4}$. The dice is equally likely to land on any of the prime numbers. Similarly, the other numbers all have the same chance as each other of being the number that the dice lands on.

(a) Find the probability that the dice lands on a prime number no more than 15 times.

[2 marks]

Question 3b

(b) Find the expected number of times the dice lands on a three.

[3 marks]

Question 3c

(c) Find the expected number of times the dice lands on a one.

[3 marks]

Question 4a

In a normal pack of 52 playing cards (without the jokers) there are 13 of each suit: hearts, clubs, diamonds and spades. James plays a card game with some friends. Each player is given nine full packs of cards. They randomly pick one card from each full pack so that they each have a hand of nine cards.

(a) Find the probability that James has at least three clubs.

[2 marks]

Question 4b

Hearts and diamonds are known as the red suits. Clubs and spades are known as the black suits.

(b) Find the probability that James only has red cards in his hand.

[3 marks]

Question 4c

(c) Find the probability that James has twice as many black cards as red cards in his hand.

[2 marks]

Question 5a

John has a collection of 12 suit jackets, of which 5 are tailored to fit him perfectly. On any given day, there is a 90% chance that John will wear a suit jacket and if he does, he chooses the suit jacket randomly.

(a) Find the probability that on given day John wears a tailored suit jacket.

[2 marks]

Question 5b

In a year John works 260 days.

(b) Find the expected number of days in a year that John wears a suit jacket.

[1 mark]

Question 5c

(c) Find the probability that John wears a non-tailored suit jacket to work at least 150 days in a year.

[3 marks]

Question 6a

Two fair dice are rolled and the numbers showing on the dice are added together. This is done 15 times and the number of times the sum is equal to 2, 7 or 11 is recorded. Let X be the discrete random variable representing the number of times that the sum is equal to 2, 7 or 11.

(a) Find the expected value of X.

[3 marks]

Question 6b

(b) Find the probability that X is at least 2 but no more than 6.

[3 marks]

Question 7a

In a game show, each contestant has seven boxes they can open. Two boxes are empty, two boxes have \$10 inside, two boxes have \$100 inside and one box has \$10 000 inside. A contestant randomly selects a box and wins the amount that is inside.

(a) Find the expected payoff from playing the game.

[2 marks]

Question 7b

Suppose there are 28 contestants.

(b) Find the expected number of players that win

(i) $10

(ii) $100

(iii) $10 000.

[3 marks]

Question 7c

(c) Find the probability that

(i) exactly four contestants win $10 000

(ii) less than six contestants win $10 000.

[4 marks]

Question 8a

Greg sells A4 paintings for $14.75 each at a market. The probability that someone at the market buys a painting from his stall is 0.06. On a given day there are 250 people at the market.

(a) Find the expected number of people that will buy a painting from Greg's stall.

[2 marks]

Question 8b

(b) Find the probability that 20 or more people buy a painting from Greg's stall.

[2 marks]

Question 8c

(c) Find the probability that Greg earns more than $118.

[4 marks]

Question 9a

Scott works as a real estate agent. The probability that he sells a house to someone over the age of 30 is 0.11 and the probability he sells a house to someone 30 or under is 0.04.

In a given week, Scott interacts with 32 potential buyers, 17 of which are over 30 and the rest are 30 or under.

(a) Find the expected number of houses that Scott sells in a week.

[3 marks]

Question 9b

(b) Find the probability Scott sells at least one house in a week.

[3 marks]

Question 10a

A multiple-choice test consists of 10 questions, where only one option is correct; six questions have three possible options and the other four questions have four possible options. Isaac takes the test and randomly selects an answer for each question.

(a) Find the expected number of questions Isaac answers correctly.

[3 marks]

Question 10b

(b) Find the probability that Isaac gets exactly two answers correct.

[5 marks]

Question 11a

A fair coin is tossed seven times.

(a) Find the probability of getting exactly five tails. Give your answer as a fraction.

[3 marks]

Question 11b

(b) Find the probability of getting exactly three heads. Give your answer as a fraction.

[3 marks]

Question 12a

On any given day, the probability that Mark rides his bike to school is $\frac{1}{3}$.

(a) Find the probability that, in a week with five school days, Mark rides his bike to school on exactly two days. Give your answer as a fraction.

[3 marks]

Question 12b

(b) Find the probability that, in a week with five school days, Mark rides his bike to school on at least one day.

[3 marks]