3.8 Vector Equations of Lines

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.8 Vector Equations of Lines
Difficulty	Very Hard

Time allowed: 110

Score: /91

Percentage: /100

Question 1

The line l has equation $r = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$ and point A has coordinates $(3,\ t,\ 2)$. Given that the shortest distance between point A and the line is $\frac{\sqrt{645}}{15}$ units, find t, where $t \in \mathbb{Z}$.

[7 marks]

Question 2a

A line l_1 has the equation $r_1 = (2+\lambda)\mathrm{i} + (6\lambda - 3)\mathrm{j} + (5 + 2\lambda)k$ and intersects the line l_2 with equation $r_2 = 5\mathrm{i} + (7 - 4\mu)\mathrm{j} + (-3 - 7\mu)k$ at point P, when $\lambda = 3$.

A third line l_3 runs parallel to l_1 and also intersects l_2 at point $X(t,\ t-2,\ -2t)$.

(a)
Find the parametric equations of l_3.

[6 marks]

Question 2b

(b)

Find the distance $|\mathrm{PX}|$.

[2 marks]

Question 3a

Consider the two intersecting lines l_1 and l_2 defined by the equations:

$$l_1 : r = \begin{pmatrix} 9 \\ 18 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} -6 \\ -3 \\ k \end{pmatrix}$$

$$l_2 : \begin{cases} x = 2\mu - 5 \\ y = -4\mu - t \\ z = 3\mu + 20 \end{cases}$$

a)

Given that the angle between l_1 and l_2 is 1.281 radians, correct to 4 significant figures, find the value of k, where $k \in \mathbb{Z}$.

[4 marks]

Question 3b

b)
Find the value of t, giving your answer correct to 3 significant figures.

[3 marks]

Question 4

Consider the two lines l_1 and l_2, where l_1 passes through the points $\mathrm{A}(11, -2, 3)$ and $\mathrm{B}(4, 4, -5)$ and l_2 is defined by the Parametric equations:

$$l_2 : \begin{cases} x = 3\mu - 7 \\ 2y = 6\mu - 9 \\ z = -4\mu - 4 \end{cases}$$

Find the shortest distance between the two lines.

[8 marks]

Question 5a

Consider the line l_1 as defined by the equation $r_1 = \begin{pmatrix} -2 \\ 5 \\ -8 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

A point $\mathrm{P}(r,\ t\ ,-r)$ lies at a distance of $\sqrt{405}$ units perpendicular from a point $\mathrm{X}(17,\ 15,-8)$ on l_1.

a)
Find all possible coordinates of P.

[6 marks]

Question 5b

b)
Given that $t > 0$, write down the set of parametric equations that defines the line l_2 that passes through points P and X.

[6 marks]

Question 6a

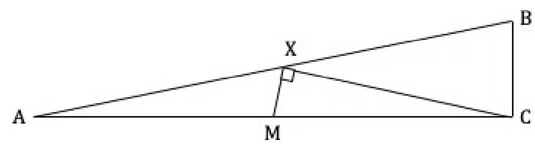
A wheelchair ramp is required to provide access to a building with a door that is located 22 cm above ground level. The maximum angle that a ramp must be from the horizontal is 4.8°.

(a)
Calculate the minimum horizontal distance that the ramp must extend out.

[2 marks]

Question 6b

The wheelchair ramp is supported by a steel frame. A cross section of the ramp can be seen in the diagram below. A metal strut joins M, the midpoint of [AC], to a point X on the line [AB]. [AB].XM=11.1 cm and $M\hat{X}C$=90°.



(b)
Using the horizontal distance found in part (a) and assuming that point A is at the origin, use a vector method to calculate the length XB.

[8 marks]

Question 7a

Some children are watching a canal boat navigating a system of locks. The boat starts at coordinates $(-10, -2, -7)$ relative to the point at which the children are standing.

The x direction is due east, the y direction is due north and the z direction is vertically upwards. All distances are measured in metres and the children are taken to be standing at the origin.

The boat travels with direction vector $1.5\mathrm{i} + 2\mathrm{j}$ for 10 metres to get into the lock and then descends vertically downwards in the lock for 11 metres before continuing along the same direction vector as it was travelling along before entering the lock.

a)
Find the coordinates of the entrance of the lock, given that the boat is now closer to the children.

[4 marks]

Question 7b

b)
Find the equation of the line along which the boat is travelling after it leaves the lock

[2 marks]

Question 7c

c)

On the next part of the journey at the point when the boat is closest to the children a child throws a flower to the boat driver. Given that the flower travels in a straight line and is caught by the boat driver, find the distance that the flower travelled.

[4 marks]

Question 8a

Consider the tetrahedron ABCD, where $\mathrm{A}(3, 5, 8)$, $\mathrm{B}(-2, 3, 2)$, $\mathrm{C}(5, -1, 3)$ and $\mathrm{D}(-3, 0, 1)$. M is the midpoint of the line BC and point P lies along the line DM.

a)

Given that the volume of the tetrahedron ABCP is $\frac{1}{3}$ of the volume of the tetrahedron ABCD, find the Vector equation of the line going through points A and P.

[4 marks]

Question 8b

X is the midpoint of $[AD]$.

b)
Find the coordinates of the point of intersection between the line found in part (a) and the line going through $[MX]$.

[5 marks]

Question 9a

An adventure park structure is made out of steel rods arranged into a frame. As a part of the structure a red rod joins the coordinates $(2, -26, 21)$ to $(-6, 14, -23)$ and a blue rod joins $(16, -33, -46)$ to $(6, -18, -21)$.

a)
Find the coordinates of the point where the red and blue rods meet each other.

[5 marks]

Question 9b

The red rod also meets a yellow rod which has the vector equation $r = (s-1)\mathrm{i} + (s-29)\mathrm{j} + (8s-3)k$. The point intersection of the red and blue rods and the red and yellow rods are joined by a taut rope.

b)

Find the length of the rope.

[4 marks]

Question 10a

A graphics designer joins the coordinates $A(1,2,3)$ to $B(1,0,1)$ and also plots the line l with parametric equations:

$$l: \begin{cases} x = 3 - 2\lambda \\ y = \lambda - 6 \\ z = 1 - \lambda \end{cases}$$

a)

Find a Vector equation of the line joining the points A and B and show that it does not intersect the line l.

[4 marks]

Question 10b

b)

Find the two possible coordinates of the point C on l such that the angle BAC is equal to $\frac{\pi}{3}$ radians.

[7 marks]