1.5 Further Proof & Reasoning

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.5 Further Proof & Reasoning
Difficulty	Hard

Time allowed: 80

Score: /64

Percentage: /100

Question 1

Prove that the equation $kx^2 - 2(k+1)x - 3k = 0$ has distinct real solutions for all values of k, where $k \in \mathbb{R}$.

[4 marks]

Question 2

Prove by mathematical induction that $9^{2n} - 1,\ n \in \mathbb{Z},\ n \geq 1$ is divisible by 16.

[4 marks]

Question 3

Prove by contradiction that $\sqrt{10}$ is irrational.

[4 marks]

Question 4

Prove by exhaustion that the sum of two consecutive square numbers between 100 and 200 is an odd number.

[4 marks]

Question 5

The three statements below are false.

In each case verify the statement is false by use of a counter example and state an alternative domain that would make the statement true.

(i)
$n^2 > 2n, \;\; n \in \mathbb{Z}^+$

(ii)
$2^n - 1$ is a prime number for $n \in N,\ 1 < n \le 4$.

(iii)
$5^n > 3^n + 4^n,\ n \in \mathbb{Z}^+$

[4 marks]

Question 6

Use mathematical induction to prove that the nth derivative of the function $f(x) = \frac{5}{x}$ is given by

$$\frac{5(-1)^n n!}{x^{(n+1)}}$$

for all integers, n, where $n \geq 1$.

[6 marks]

Question 7

Prove that $a^2 - 8b - 11 \neq 0$ if $a,\ b \in \mathbb{Z}$.

[6 marks]

Question 8

The product of three consecutive integers is added to the middle integer.

Prove that the result is a perfect cube.

[4 marks]

Question 9

Prove by mathematical induction, that for $n \in \mathbb{Z}^+$,

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}$$

[6 marks]

Question 10

Use a contradiction to prove that the difference between a rational number and an irrational number is irrational.

[6 marks]

Question 11

Prove that there are no non-zero real values of a and b such that $(a + bi)^2 = a + bi$.

[4 marks]

Question 12

Prove by mathematical induction that if $f(x) = xe^{2x}$ then $f^{(n)}(x) = (2^n x + n2^{n-1})e^{2x}$.

[6 marks]

Question 13

Prove by mathematical induction that

$$(\cos\theta - \mathrm{i}\sin\theta)^n = \cos(n\theta) - \mathrm{i}\sin(n\theta), \text{ for all } n \in \mathbb{Z}^+$$

[6 marks]