IB Physics DP

YOUR NOTES

6. Circular Motion & Gravitation

CONTENTS

6.1 Circular Motion

6.1.1 Circular Motion

YOUR NOTES

Properties of Circular Motion

- For an object moving in a circle, it will have the following properties:
  - Period
  - Frequency
  - Angular displacement
  - Angular velocity
- These properties can be inferred from the properties of objects moving in a straight line combined with the geometry of a circle

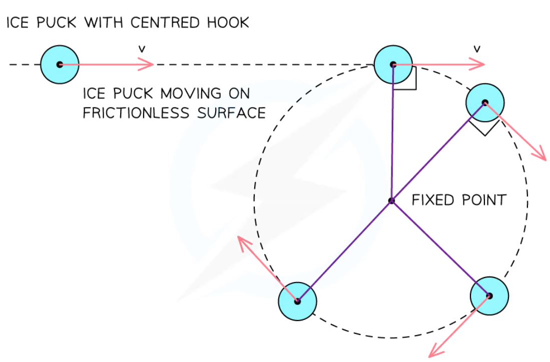
Motion in a Straight Line

- When an object moves in a straight line at a constant speed its motion can be described as follows:
  - The object moves at a constant velocity, *v*
  - Constant velocity means zero acceleration, *a*
  - Newton's First Law of motion says the object will continue to travel in a straight line at a constant speed unless acted on by another force
  - Newton's Second Law of motion says for zero acceleration that there is no net or resultant force
- For example, an ice hockey puck moving across a flat frictionless ice rink

An ice puck moving in a straight line

Motion in a Circle

- If one end of a string was attached to the puck, and the other attached to a fixed point, it would no longer travel in a straight line, it would begin to travel in a circle



The red arrows represent the velocity vectors of the puck. If the string were cut, the puck would move off in the direction shown by the red vector, as predicted by Newton's first law.

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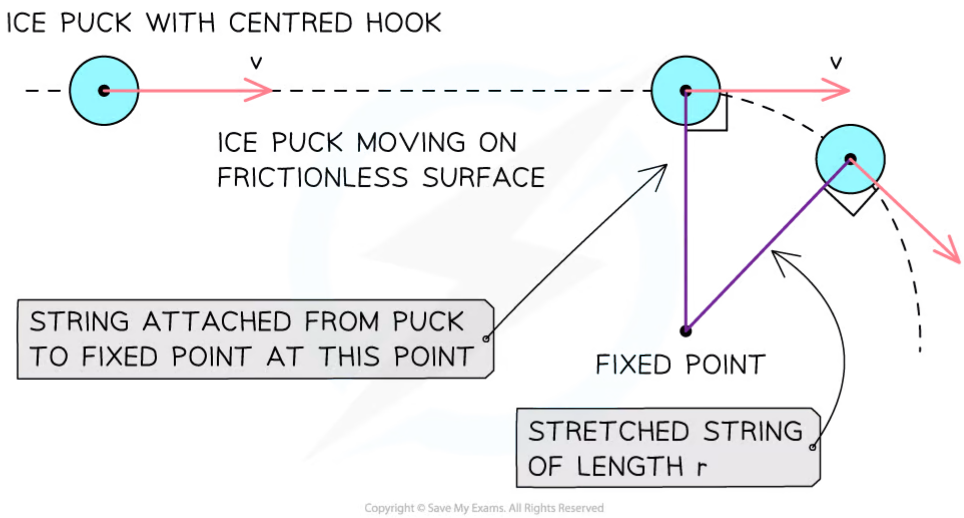
- The motion of the puck can now be described as follows:
  - As the puck moves it stretches the string a little to a length *r*
  - The stretched string applies a force to the puck pulling it so that it moves in a circle of radius *r* around the fixed point
- The force acts at 90° to the velocity so there is no force component in the direction of velocity
  - As a result, the **magnitude** of the velocity is constant
  - However, the **direction** of the velocity **changes**
- As it starts to move in a circle the tension of the string continues to pull the puck at 90° to the velocity
  - The speed does not change, hence, this is called **uniform circular motion**



The applied force (tension) from the string causes the puck to move with uniform circular motion

Time Period & Frequency

- If the circle has a radius *r*, then the distance through which the puck moves as it completes one rotation is equal to the circumference of the circle = $2\pi r$
- The speed of the puck is therefore equal to:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2\pi r}{T}$$

- Where:
  - r = the radius of the circle (m)
  - T = the time period (s)
- This is the same as the time period in waves and simple harmonic motion (SHM)

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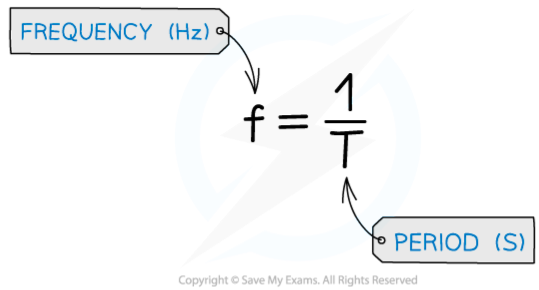
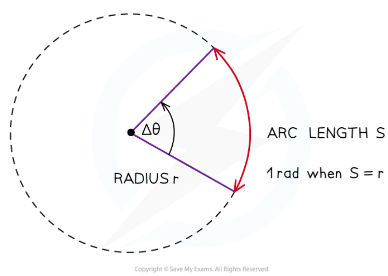
- The frequency, f, can be determined from the equation:



Angles in Radians

- A **radian** (rad) is defined as:

    The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle



When the angle is equal to one radian, the length of the arc (S) is equal to the radius (r) of the circle

- Radians are commonly written in terms of π
- The angle in radians for a complete circle (360°) is equal to:

$$\frac{\text{circumference of circle}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$$

- Use the following equation to convert from degrees to radians:

$$\theta° \times \frac{\pi}{180} = \theta \text{ rad}$$

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Table of common degrees to radians conversions

Degrees (°)	Radians (rads)
360	2π
270	$\frac{3\pi}{2}$
180	π
90	$\frac{\pi}{2}$

Angular Displacement

- In circular motion, it is more convenient to measure angular displacement in units of radians rather than units of degrees
- Angular displacement is defined as:

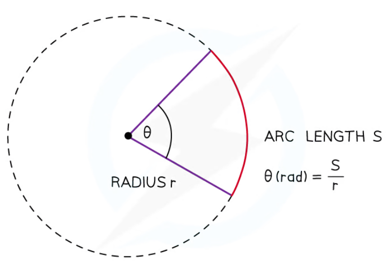
The change in angle, in radians, of a body as it rotates around a circle

- This can be summarised in equation form:

$$\Delta\theta = \frac{\text{distance travelled around the circle}}{\text{radius of the circle}} = \frac{S}{r}$$

- Where:
  - $\Delta\theta$ = angular displacement, or angle of rotation (radians)
  - S = length of the arc, or the distance travelled around the circle (m)
  - r = radius of the circle (m)

- Note: both distances must be measured in the same units e.g. metres



An angle in radians, subtended at the centre of a circle, is the arc length divided by the radius of the circle

Angular Speed

- Any object rotating with a uniform circular motion has a constant speed but constantly changing velocity
- Its velocity is changing so it is **accelerating**

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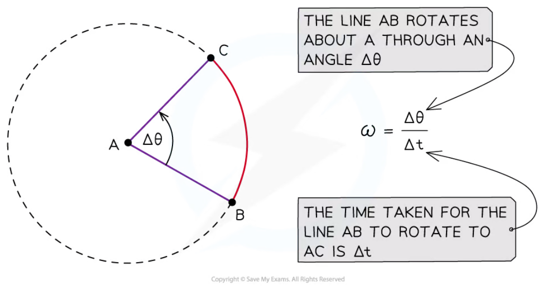
- But at the same time, it is moving at a constant speed

- The angular speed, ω, of a body in circular motion is defined as:

  The rate of change in angular displacement with respect to time

- Angular speed is a **scalar** quantity and is measured in rad s^{-1}

- The angular speed does not depend on the length of the line AB
- The line AB will sweep out an angle of 2π rad in a time T

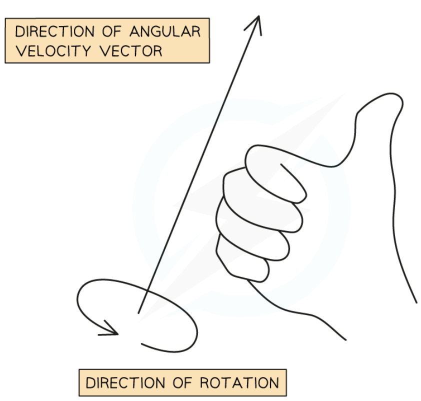


The angular speed is ω is the rate at which the line AB rotates

Angular Velocity

- Angular velocity is a **vector** quantity and is measured in rad s^{-1}
- Angular speed is the **magnitude** of the angular velocity
- The direction of the angular velocity vector points along the axis of rotation but depends on the direction of rotation

- The angular velocity vector points in the direction a corkscrew moves when it rotates in the same direction as the circular motion



Wrap the right hand around the axis of rotation so that the fingers are pointing in the direction of rotation. The thumb points in the direction of the angular velocity vector

Equation Linking Linear & Angular Speed

- The angular speed and velocity don't depend on the radius of the circle
- The linear speed does depend on the radius of the circle

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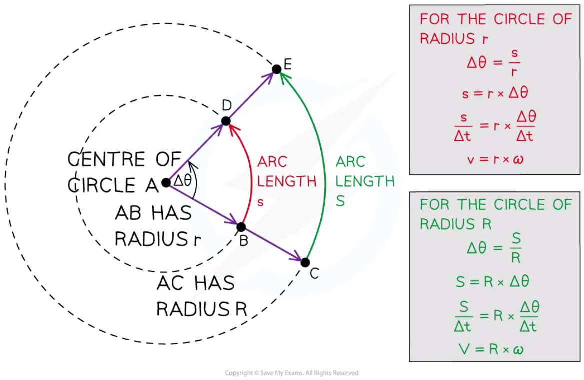


The angle Δθ is swept out in a time Δt, but the arc lengths s and S are different and so are the linear speeds

- The linear speed, *v*, is related to the angular speed, ω, by the equation:

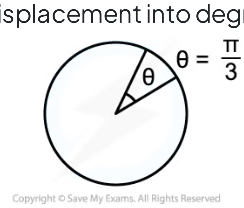
$$v = r\omega$$

- Where:
  - v = linear speed (m s^{-1})
  - r = radius of circle (m)
  - ω = angular speed (rad s^{-1})
- Taking the angular displacement of a complete cycle as 2π, the angular speed ω can be calculated using the equation:

$$\omega = \frac{v}{r} = 2\pi f = \frac{2\pi}{T}$$

Worked Example

Convert the following angular displacement into degrees:



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STEP 1

REARRANGE DEGREES TO RADIANS CONVERSION EQUATION

DEGREES ⟶ RADIANS $\quad \theta^\circ \times \frac{\pi}{180} = \theta \text{ RAD}$

RADIANS ⟶ DEGREES $\quad \theta \text{ RAD} \times \frac{180}{\pi} = \theta^\circ$

STEP 2

SUBSTITUTE VALUE

π's WILL CANCEL OUT

$$\frac{\pi}{3} \text{ RAD} \times \frac{180}{\pi} = \frac{180^\circ}{3} = 60^\circ$$

Worked Example

A bird flies in a horizontal circle with an angular speed of 5.25 rad s^{-1} of radius 650 m.

Calculate:

1. The linear speed of the bird
2. The frequency of the bird flying in a complete circle

a) STEP 1

LINEAR SPEED EQUATION

$v = r\omega$

STEP 2

SUBSTITUTE IN VALUES

$V = 650 \times 5.25 = 3412.5 = 3410 \text{ ms}^{-1}$ (3 s.f.)

b) STEP 1

ANGULAR SPEED WITH FREQUENCY EQUATION

$\omega = 2\pi f$

STEP 2

REARRANGE FOR FREQUENCY

$$f = \frac{\omega}{2\pi}$$

STEP 3

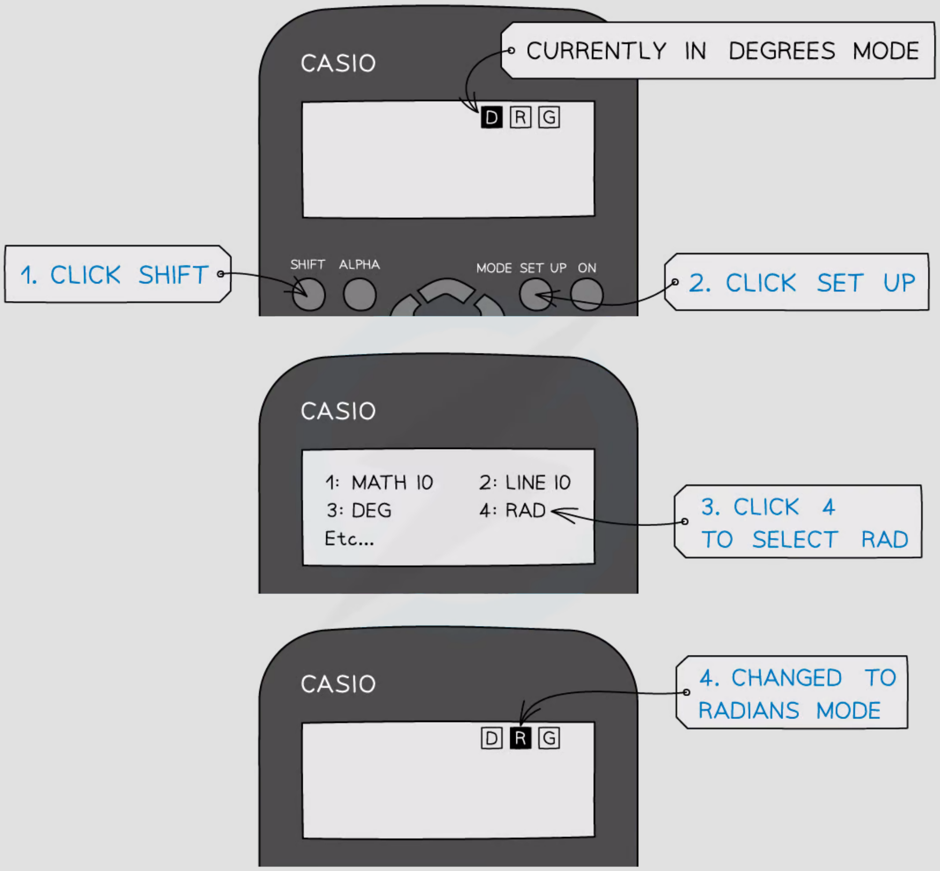
SUBSTITUTE IN VALUES

$$f = \frac{5.25}{2\pi} = 0.83556... = 0.836 \text{ Hz} \quad (3 \text{ s.f.})$$

YOUR NOTES

Exam Tip

You will notice your calculator has a degree (Deg) and radians (Rad) modeThis is shown by the "D" or "R" highlighted at the top of the screenRemember to make sure it's in the right mode when using **trigonometric** functions (sin, cos, tan) depending on whether the answer is required in **degrees** or **radians**It is extremely common for students to get the wrong answer (and lose marks) because their calculator is in the wrong mode - make sure this doesn't happen to you!



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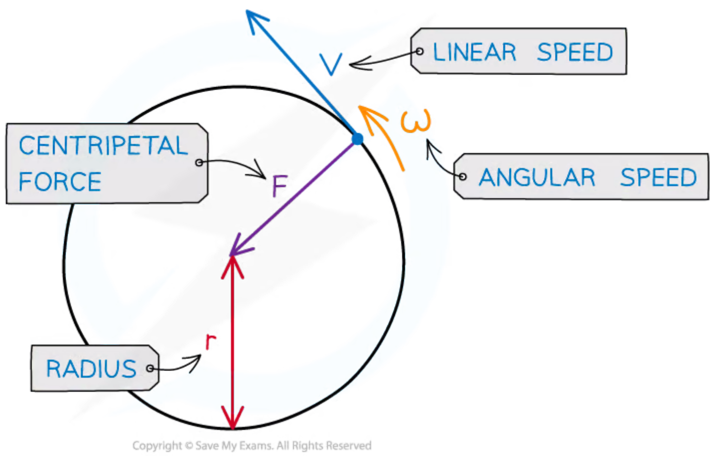
Centripetal Force

- An object moving in a circle is not in equilibrium, it has a resultant force acting upon it
  - This is known as the **centripetal force** and is what keeps the object moving in a circle
- The centripetal force (*F*) is defined as:

The resultant force perpendicular to the velocity, and therefore directed towards the centre of the circle, required to keep a body in uniform circular motion

- The magnitude of the centripetal force *F* can be calculated using:

$$F = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$



Centripetal force is always perpendicular to the linear velocity (i.e., the direction of travel)

- Where:
  - F = centripetal force (N)
  - v = linear speed (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)
- **Note:** centripetal force and centripetal acceleration act in the **same direction**
  - This is due to Newton's Second Law
- The centripetal force is **not** a separate force of its own
- It can be any type of force, depending on the situation, which keeps an object moving in a circular path
  - For example, tension, friction, gravitational, electrical or magnetic

Examples of centripetal force

Situation	Centripetal force
Car travelling around a roundabout	Friction between car tyres and the road
Ball attached to a rope moving in a circle	Tension in the rope
Earth orbiting the Sun	Gravitational force

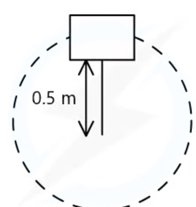
- When solving circular motion problems involving one of these forces, the equation for centripetal force can be equated to the relevant force equation
- For example, for a charged particle travelling in a circle, the **centripetal force** causing the charged particle to move in a circle is provided by the **magnetic force**
- Therefore, equating the expressions for centripetal force and magnetic force gives the following:

$$\frac{mv^2}{r} = Bqv$$

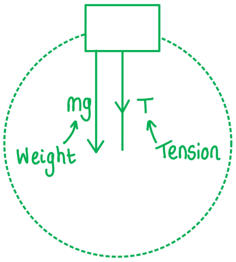
- Where:
  - B = magnetic field strength (T)
  - q = charge on the particle (C)
  - m = mass of the particle (kg)
  - v = speed of the particle (m s^{-1})
  - r = radius of orbit (m)

Worked Example

A bucket of mass 8.0 kg is filled with water is attached to a string of length 0.5 m.What is the minimum speed the bucket must have at the top of the circle so no water spills out?



Step 1: Draw the forces on the bucket at the top



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Step 2: Calculate the centripetal force

- The weight of the bucket = mg
- This is equal to the centripetal force since it is directed towards the centre of the circle

$$mg = \frac{mv^2}{r}$$

Step 3: Rearrange for velocity *v*

- *m* cancels from both sides

$$v = \sqrt{gr}$$

Step 4: Substitute in values

$$v = \sqrt{9.81 \times 0.5} = 2.21 \text{ m s}^{-1}$$

6.1.3 Centripetal Acceleration

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Centripetal Acceleration

- For an object moving in a circle:
  - The acceleration is towards the centre of the circle
  - The magnitude of the centripetal acceleration *a* is:

$$a = \frac{v^2}{r}$$

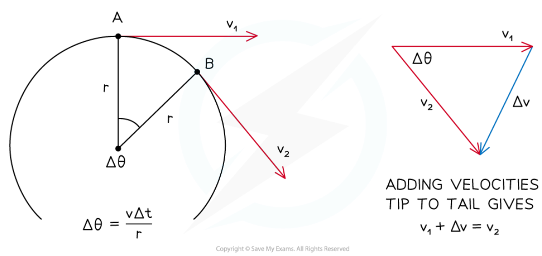
- Where:
  - a = centripetal acceleration (m s^{-2})
  - v = linear speed (m s^{-1})
  - r = radius of orbit (m)

- Uniform circular motion is **continuously changing direction,** and therefore is **constantly changing velocity**
  - The object must therefore be **accelerating**

- This is called the **centripetal acceleration**

Direction of the Centripetal Acceleration

- The **centripetal acceleration** is **perpendicular** to the direction of the **linear velocity**
  - Centripetal means it acts **towards the centre** of the circular path



Slide a ruler parallel to Δv towards the circle. Midway between A and B, Δv points towards the centre of the circle. This is the same direction as the centripetal acceleration

- If an object moves through a section of a circle during some time Δt
- The change in velocity during this time is Δv
- The centripetal acceleration is Δv (a vector) divided by Δt (a scalar)
  - The centripetal acceleration points in the same direction as the **change** in velocity Δv

- The centripetal acceleration is caused by a **centripetal force** of constant magnitude that also acts **perpendicular** to the direction of motion (towards the centre)
- There is no component of the centripetal force in the direction of the velocity
  - Therefore, there is no acceleration in the direction of the velocity
  - Hence, there is uniform motion at constant speed

- Therefore, the centripetal acceleration and force act in the **same direction**

Magnitude of the Centripetal Acceleration

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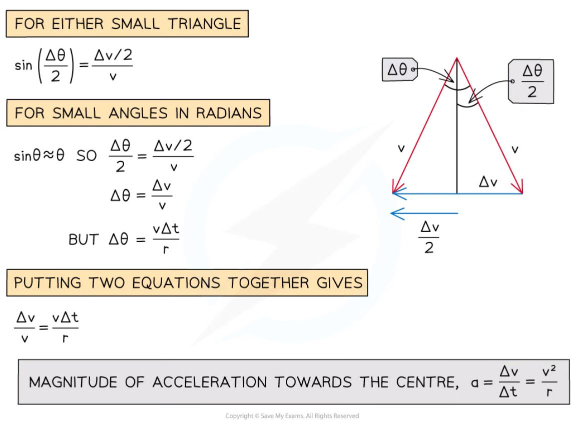
- In the diagram above notice how the angle $\Delta\theta$ is defined in terms of the arc length $v\Delta t$ and the radius r
- v is the magnitude of v_1 **and** v_2
- Notes for deriving the equation for centripetal acceleration:
  - The vector triangle should be formed so that Δv is horizontal
  - The velocity vectors v should be of the same length
  - Hence, the vertical line bisects the angle $\Delta\theta$ and the vector Δv
  - Use trigonometry for one of the small triangles
  - The small-angle approximation requires that the angles are in radians
  - The two equations for $\Delta\theta$ lead to the magnitude of the centripetal acceleration



Deriving the equation for the magnitude of the centripetal acceleration

- This leads to the equation for centripetal acceleration:

$$a = \frac{v^2}{r}$$

- Using the equation relating angular speed ω and linear speed v:

$$v = r\omega$$

- These equations can be combined to give another form of the centripetal acceleration equation:

$$a = \frac{(r\omega)^2}{r}$$

$$a = r\omega^2$$

- Where:
  - a = centripetal acceleration (m s^{-2})
  - v = linear speed (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)
- Uniform centripetal acceleration is defined as:

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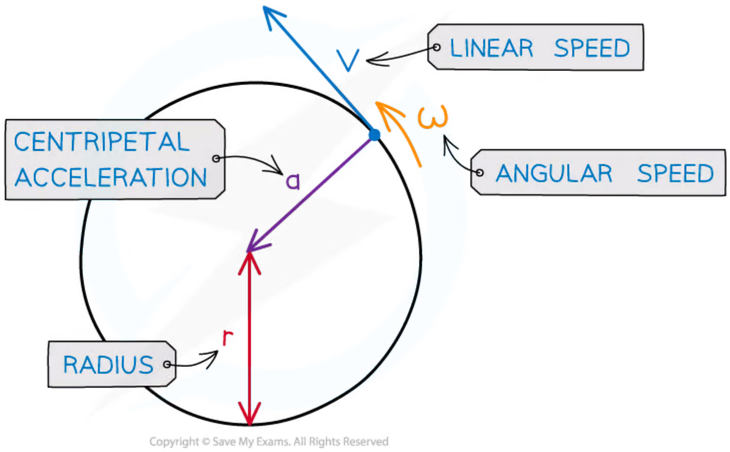
The acceleration of an object towards the centre of a circle when an object is in motion (rotating) around a circle at a constant speed



Centripetal acceleration is always directed toward the centre of the circle and is perpendicular to the object's velocity

Worked Example

A domestic washing machine has a spin cycle of 1200 rpm (revolutions per minute) and a diameter of 50 cm.

Calculate the centripetal acceleration experienced by the washing during the spin cycle.

Step 1: List the known quantities

- Radius of the drum, r = ½ × 50 cm = 25 cm

Step 2: Convert the revolutions per minute to revolutions per second

$$1200 \div 60 = 20 \text{ rev s}^{-1}$$

Step 3: Convert revolutions per second to angular speed in radians per second

$$1 \text{ rev s}^{-1} = 2\pi \text{ rad s}^{-1}$$

$$20 \text{ rev s}^{-1} = 40\pi \text{ rad s}^{-1} = \omega$$

Step 4: Write the equation linking centripetal acceleration and angular speed

$$a = r\omega^2$$

Step 5: Calculate the centripetal acceleration

$$a = (25 \times 10^{-2}) \times (40\pi)^2$$

Step 6: State the final answer

$$\mathbf{a = 3900 \text{ m s}^{-2} \text{ (2 s.f.)}}$$

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Worked Example

A ball tied to a string is rotating in a horizontal circle with a radius of 1.5 m and an angular speed of 3.5 rad s^{-1}.

Calculate its centripetal acceleration if the radius was twice as large and angular speed was twice as fast.

STEP 1

ANGULAR ACCELERATION EQUATION WITH ANGULAR SPEED

$a = r\omega^2$

STEP 2

CHANGE IN ANGULAR ACCELERATION WITH TWICE THE RADIUS AND ANGULAR SPEED

$a = (2r) \times (2\omega)^2 = 2r \times 4\omega^2 = 8r\omega^2$

THE CENTRIPETAL ACCELERATION WILL BE 8x BIGGER

STEP 3

SUBSTITUTE IN VALUES OF RADIUS AND ANGULAR SPEED

$a = 8r\omega^2 = 8 \times 1.5 \times 3.5^2 = 147 \text{ ms}^{-2}$

Exam Tip

The key takeaways for an object moving in a circle are:

- The **magnitude** of the velocity vector does **not** change
- The **direction** of the velocity vector **does** change
- Therefore, there is an **acceleration** despite the speed not changing

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Applications of Circular Motion

Horizontal Circular Motion

- An example of horizontal circular motion is a vehicle driving on a curved road
- The forces acting on the vehicle are:
  - The **friction** between the tyres and the road
  - The **weight** of the vehicle downwards
- In this case, the centripetal force required to make this turn is provided by the frictional force
  - This is because the force of friction acts towards the centre of the circular path
- Since the centripetal force is provided by the force of friction, the following equation can be written:

$$\frac{mv^2}{r} = \mu mg$$

- Where:
  - m = mass of the vehicle (kg)
  - v = speed of the vehicle (m s^{-1})
  - r = radius of the circular path (m)
  - μ = static coefficient of friction
  - g = acceleration due to gravity (m s^{-2})
- Rearranging this equation for v gives:

$$v^2 = \mu gr$$

$$v_{max} = \sqrt{\mu gr}$$

- This expression gives the maximum speed at which the vehicle can travel around the curved road without skidding
  - If the speed exceeds this, then the vehicle is likely to skid
  - This is because the centripetal force required to keep the car in a circular path could not be provided by friction, as it would be too large

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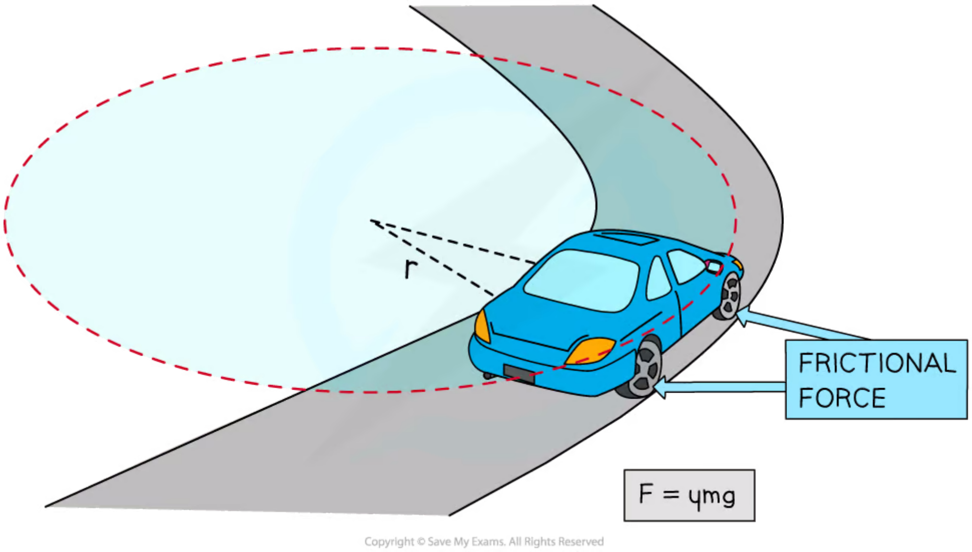

- Therefore, in order for a vehicle to avoid skidding on a curved road of radius *r*, its speed must satisfy the equation

$$v < \sqrt{\mu g r}$$

Banking

- A banked road, or track, is a curved surface where the outer edge is raised higher than the inner edge
  - The purpose of this is to make it safer for vehicles to travel on the curved road, or track, at a reasonable speed without skidding
- When a road is banked, the centripetal force no longer depends on the friction between the tyres and the road
- Instead, the centripetal force depends solely on the normal force and the weight of the vehicle

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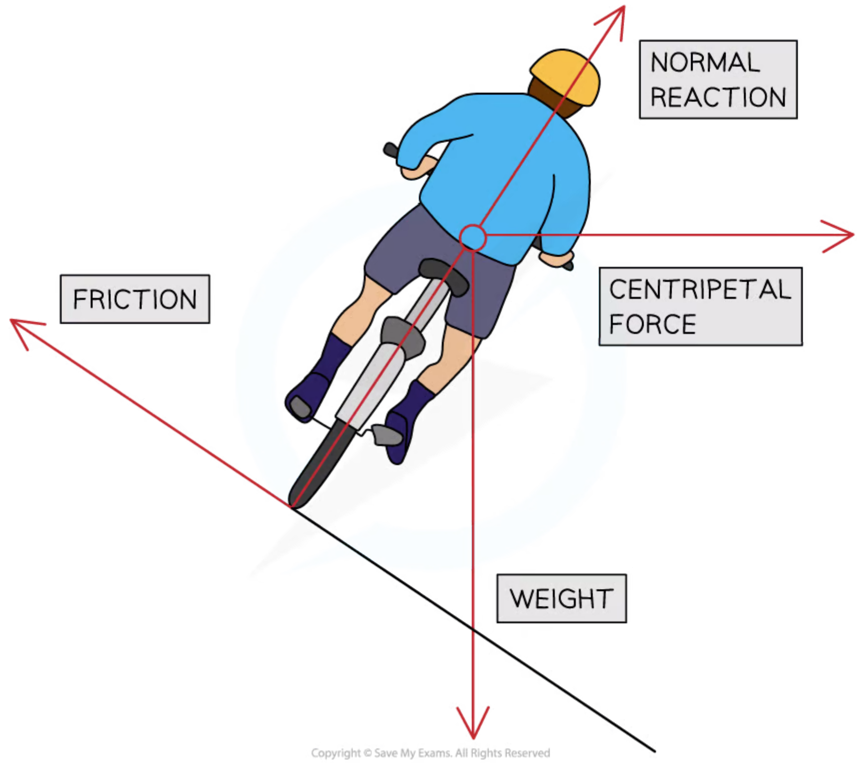

Vertical Circular Motion

- An example of vertical circular motion is swinging a ball on a string in a vertical circle
- The forces acting on the ball are:
  - The **tension** in the string
  - The **weight** of the ball downwards
- As the ball moves around the circle, the **direction** of the tension will change continuously
- The **magnitude** of the tension will also vary continuously, reaching a **maximum** value at the **bottom** and a **minimum** value at the **top**
  - This is because the direction of the weight of the ball never changes, so the resultant force will vary depending on the position of the ball in the circle

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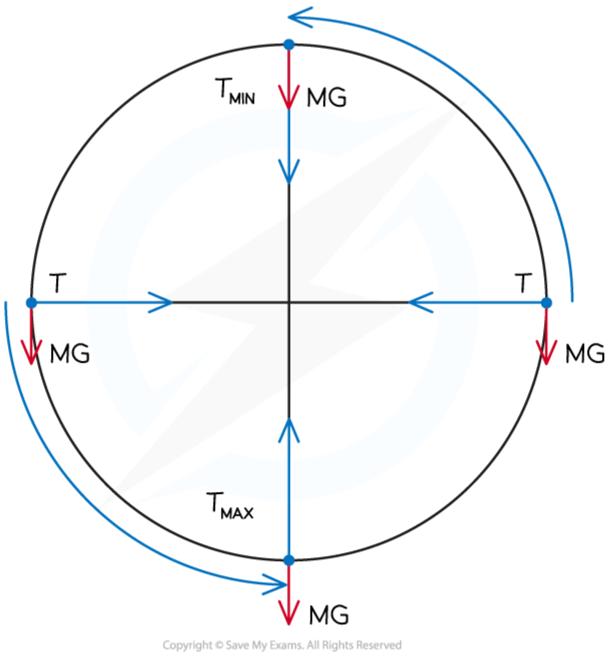


- At the bottom of the circle, the tension must overcome the weight, this can be written as:

$$T_{max} = \frac{mv^2}{r} + mg$$

- As a result, the acceleration, and hence, the **speed** of the ball will be **slower** at the top
- At the top of the circle, the tension and weight act in the same direction, this can be written as:

$$T_{min} = \frac{mv^2}{r} - mg$$

- As a result, the acceleration, and hence, the **speed** of the ball will be **faster** at the bottom

Exam Tip

You do not need to know the mathematics of banking but you may be required to explain the principles unpinning it, so make sure you understand it!

YOUR NOTES

6.2 Newton's Law of Gravitation

6.2.1 Newton's Law of Gravitation

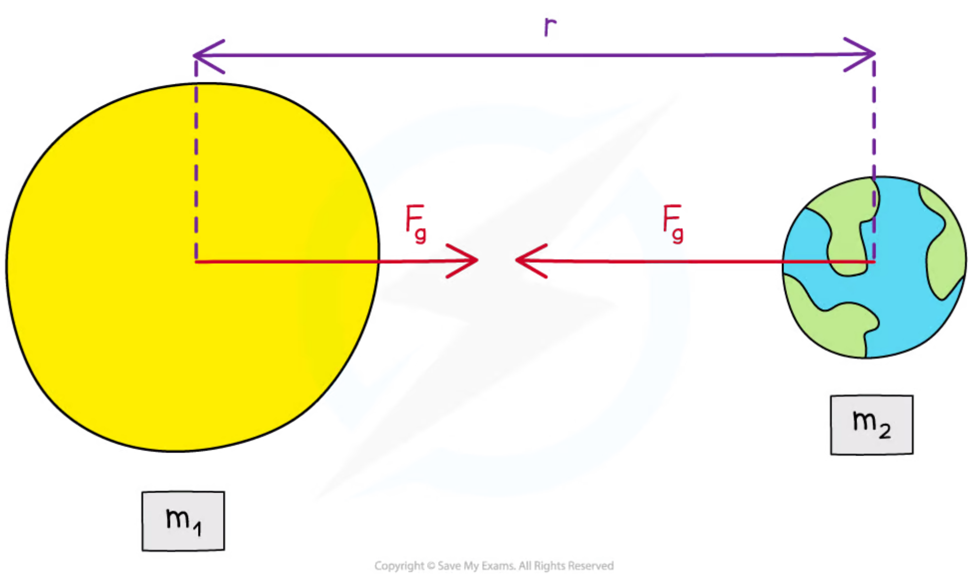
Newton's Law of Gravitation

- The gravitational force between two bodies outside a uniform field, e.g. between the Earth and the Sun, is defined by Newton's Law of Gravitation
  - Recall that the mass of a uniform sphere can be considered to be a point mass at its centre
- Newton's Law of Gravitation states that:

The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation

- In equation form, this can be written as:

$$F = G\frac{Mm}{r^2}$$



The gravitational force between two masses outside a uniform field is defined by Newton's Law of Gravitation

- Where:
  - F = gravitational force between two masses (N)
  - G = Newton's Gravitational Constant
  - m and M = two points masses (kg) (These are sometimes labelled m_1 and m_2)
  - r = distance between the centre of the two masses (m)

- Although planets are not point masses, their separation is much larger than their radius

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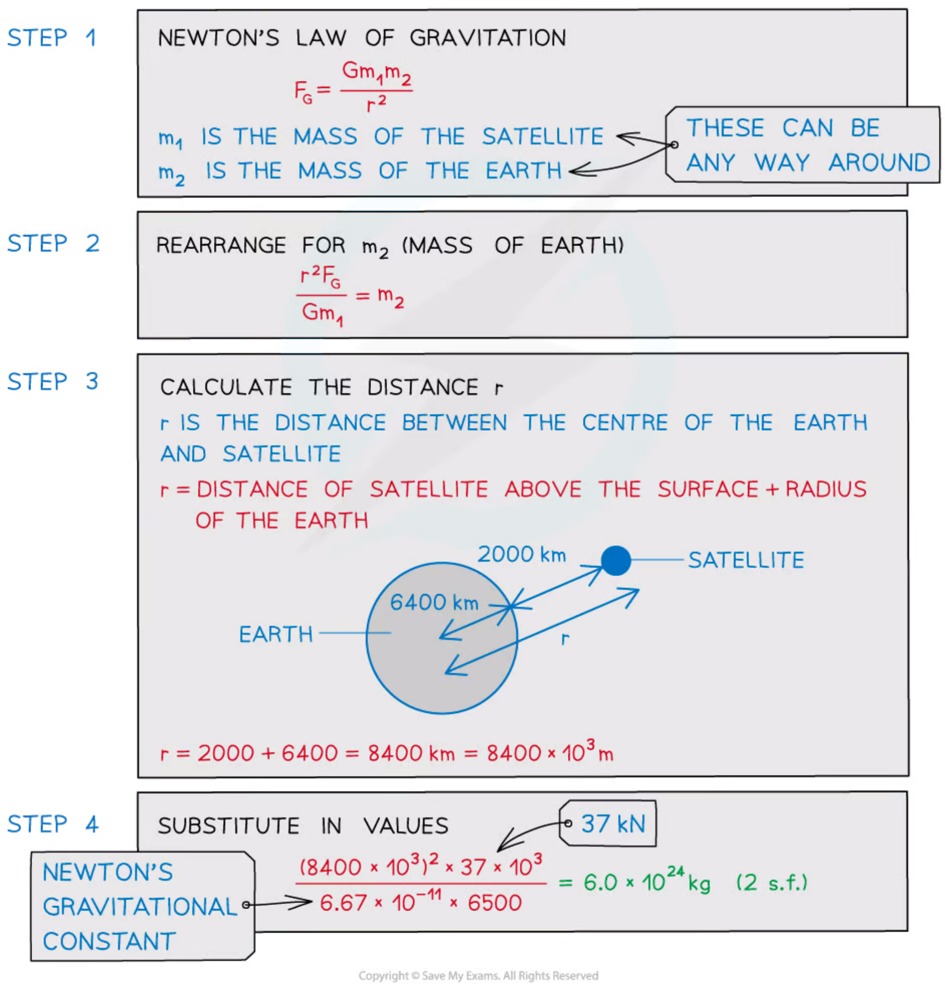
- Therefore, Newton's law of gravitation applies to planets orbiting the Sun

- The $1/r^2$ relation is called the 'inverse square law'
- This means that when a mass is twice as far away from another, its force due to gravity reduces by $(½)^2 = ¼$

Worked Example

A satellite with mass 6500 kg is orbiting the Earth at 2000 km above the Earth's surface. The gravitational force between them is 37 kN.

Calculate the mass of the Earth. (Radius of the Earth = 6400 km)



Exam Tip

A common mistake in exams is to forget to **add together** the distance from the surface of the planet and its radius to obtain the value of *r*. The distance *r* is measured from the **centre** of the mass, which is from the **centre** of the planet.

6.2.2 Circular Orbits

YOUR NOTES

Circular Orbits

- Since most planets and satellites have a near circular orbit, the gravitational force F_G between the sun or another planet provides the centripetal force needed to stay in an orbit
- Both the gravitational force and centripetal force are **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass *m* orbiting Earth with mass *M* at a distance *r* from the centre travelling with linear speed *v*

$$F_G = F_{circ}$$

- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

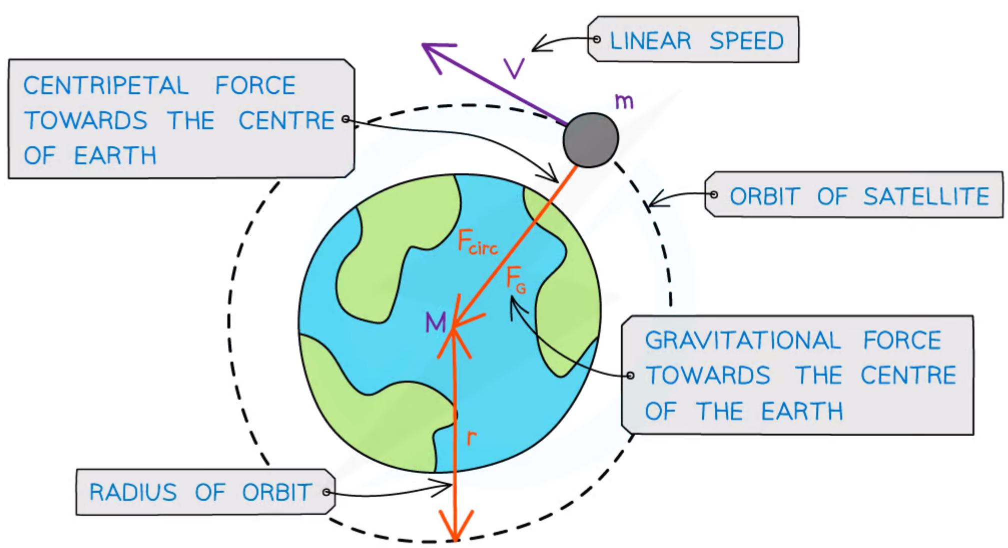
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The mass of the satellite *m* will cancel out on both sides to give:

$$v^2 = \frac{GM}{r}$$

- Where:
  - v = linear speed of the mass in orbit (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object being orbited (kg)
  - r = orbital radius (m)

- This means that all satellites, whatever their mass, will travel at the same speed *v* in a particular orbit radius *r*
- Recall that since the direction of a planet orbiting in circular motion is constantly changing, it has **centripetal acceleration**



A satellite in orbit around the Earth travels in circular motion

YOUR NOTES
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Time Period & Orbital Radius Relation

- Since a planet or a satellite is travelling in circular motion when in order, its orbital time period *T* to travel the circumference of the orbit $2\pi r$, the linear speed *v* is:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time and first introduced in the circular motion topic
- Substituting the value of the linear speed *v* from equating the gravitational and centripetal force into the above equation gives:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

- Squaring out the brackets and rearranging for T^2 gives the equation relating the time period *T* and orbital radius *r*:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

- Where:
  - T = time period of the orbit (s)
  - r = orbital radius (m)
  - G = Newton's Gravitational Constant
  - M = mass of the object being orbited (kg)

- The equation shows that the orbital period *T* is related to the radius *r* of the orbit. This is also known as Kepler's third law:

  For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit

- Kepler's third law can be summarised as:

$$T^2 \propto r^3$$

Graphical Representation of $T^2 \propto r^3$

- The relationship between *T* and *r* can be shown using a logarithmic plot
- Plotting of *T* in years against *r* in AU (astronomical units) (for the planets in our solar system) on a log paper or taking logs and plotting on regular graph paper is a straight-line graph:
- The graph does not go through the origin since it has a negative y-intercept

YOUR NOTES
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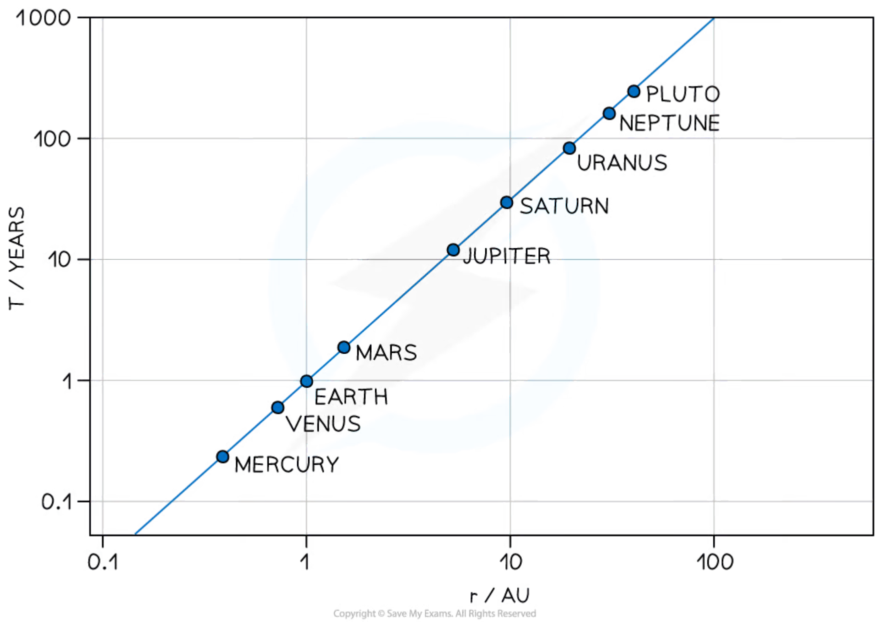
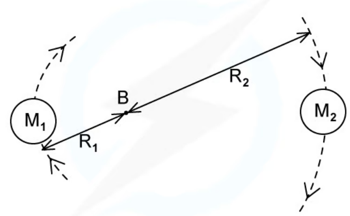


Maths Tip

- The ∝ symbol means 'proportional to'
- Find out more about proportional relationships between two variables in the "proportional relationships " section of the A Level Maths revision notes

Worked Example

A binary star system constant of two stars orbiting about a fixed point **B**.The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 has a radius of R_2. Both have linear speed v and an angular speed ω about **B**.



State the following formula, in terms of G, M_2, R_1 and R_2

(i) The angular speed ω of M_1

(ii) The time period T for each star in terms of angular speed ω

YOUR NOTES

(i) The angular speed ω of M_1

Step 1: Equate the centripetal force to the gravitational force

$$M_1R_1\omega^2 = \frac{GM_1M_2}{(R_1+R_2)^2}$$

Step 2: M_1 cancels on both sides

$$R_1\omega^2 = \frac{GM_2}{(R_1+R_2)^2}$$

Step 3: Rearrange for angular velocity ω

$$\omega^2 = \frac{GM_2}{R_1(R_1+R_2)^2}$$

Step 4: Square root both sides

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}}$$

(ii) The time period T for each star in terms of angular speed ω

Step 1: Write down the angular speed ω equation with time period T

$$\omega = \frac{2\pi}{T}$$

Step 2: Rearrange for T

$$T = \frac{2\pi}{\omega}$$

Step 3: Substitute in ω from part (i)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{GM_2}}$$

YOUR NOTES

Exam Tip

Many of the calculations in the Gravitation questions depend on the equations for circular motion. Be sure to revisit these and understand how to use them! You will be expected to remember the derivation for $T^2 \propto r^3$ relation, so make sure you understand each step

YOUR NOTES
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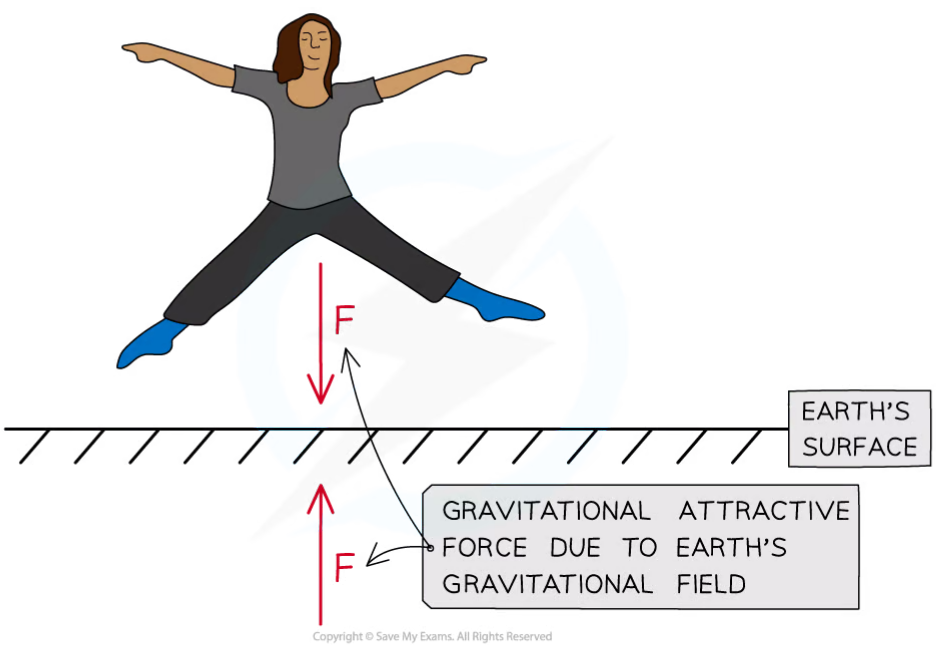
Gravitational Field Strength

- There is a universal force of attraction between all matter with **mass**
  - This force is known as the 'force due to gravity' or the **weight**
- The Earth's gravitational field is responsible for the weight of all objects on Earth
- A gravitational field is defined as:

  A region of space where a test mass experiences a force due to the gravitational attraction of another mass

- The direction of the gravitational field is always towards the centre of the mass causing the field
  - Gravitational forces **cannot** be repulsive
- Gravity has an infinite range, meaning it affects all objects in the universe
  - There is a **greater** gravitational force around objects with a **large mass** (such as planets)
  - There is a **smaller** gravitational force around objects with a **small mass** (almost negligible for atoms)



The Earth's gravitational field produces an attractive force. The force of gravity is always attractive

- The gravitational field strength at a point is defined as:

  The force per unit mass experienced by a test mass at that point

- This can be written in equation form as:

YOUR NOTES
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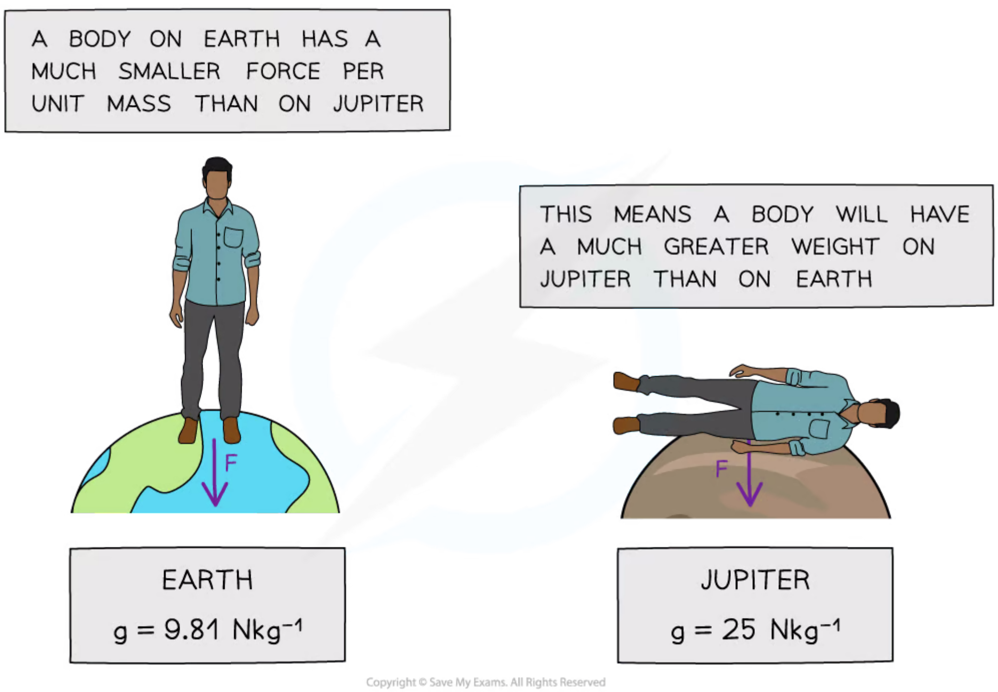
$$g = \frac{F}{m}$$

- Where:
  - g = gravitational field strength ($N\ kg^{-1}$)
  - F = force due to gravity, or weight (N)
  - m = mass of test mass in the field (kg)
- This equation shows that:
  - On planets with a large value of g, the gravitational force per unit mass is **greater** than on planets with a smaller value of g
- An object's mass remains the **same** at all points in space
  - However, on planets such as Jupiter, the **weight** of an object will be greater than on a less massive planet, such as Earth
  - This means the gravitational force would be so high that humans, for example, would not be unable to fully stand up



A person's weight on Jupiter would be so large that a human would be unable to fully stand up

- Factors that affect the gravitational field strength at the surface of a planet are:
  - The **radius** r (or diameter) of the planet
  - The **mass** M (or density) of the planet
- This can be shown by equating the equation $F = mg$ with Newton's law of gravitation:

$$F = \frac{GMm}{r^2}$$

YOUR NOTES
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- Substituting the force *F* with the gravitational force *mg* leads to:

$$mg = \frac{GMm}{r^2}$$

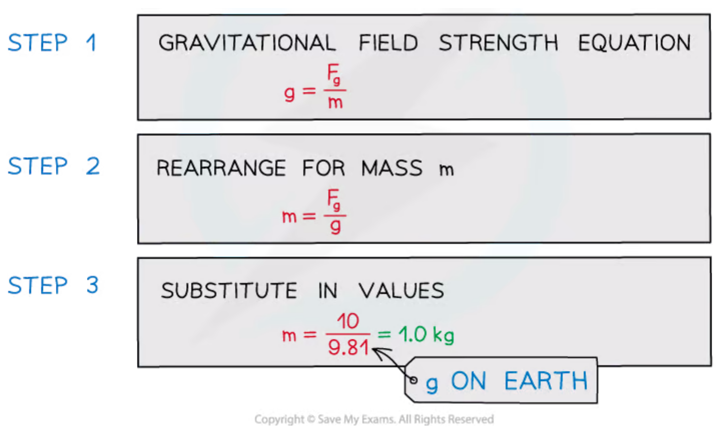
- Cancelling the mass of the test mass, *m*, leads to the equation:

$$g = \frac{GM}{r^2}$$

- Where:
  - *G* = Newton's Gravitational Constant
  - *M = mass of the body causing the field (kg)*
  - *r* = distance from the mass where you are calculating the field strength (m)
- This equation shows that:
  - The gravitational field strength *g* depends only on the mass of the body *M* causing the field
  - Hence, objects with any mass *m* in that field will experience the **same gravitational field strength**
  - The gravitational field strength *g* is **inversely proportional** to the **square** of the radial distance, r^2

Worked Example

Calculate the mass of an object with weight 10 N on Earth.



Worked Example

The mean density of the Moon is 3/5 times the mean density of the Earth. The gravitational field strength on the Moon is 1/6 of the value on Earth.

Determine the ratio of the Moon's radius r_M and the Earth's radius r_E.

YOUR NOTES
⬇

Step 1: Write down the known quantities

$$\rho_M = \frac{3}{5}\rho_E$$
$$g_M = \frac{1}{6}g_E$$

g_M = gravitational field strength on the Moon, ρ_M = mean density of the Moon

g_E = gravitational field strength on the Earth, ρ_E = mean density of the Earth

Step 2: The volumes of the Earth and Moon are equal to the volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

Step 3: Write the density equation and rearrange for mass M

$$\rho = \frac{M}{V}$$

$$M = \rho V$$

Step 4: Write the gravitational field strength equation

$$g = \frac{GM}{r^2}$$

Step 5: Substitute M in terms of ρ and V

$$g = \frac{G\rho V}{r^2}$$

Step 6: Substitute the volume of a sphere equation for V, and simplify

$$g = \frac{G\rho 4\pi r^3}{3r^2} = \frac{G\rho 4\pi r}{3}$$

Step 7: Find the ratio of the gravitational field strength

$$\frac{g_M}{g_E} = \frac{G\rho_M 4\pi r_M}{3} \div \frac{G\rho_E 4\pi r_E}{3} = \frac{\rho_M r_M}{\rho_E r_E}$$

Step 8: Rearrange and calculate the ratio of the Moon's radius r_M and the Earth's radius r_E

$$\frac{r_M}{r_E} = \frac{\rho_E g_M}{\rho_M g_E} = \frac{\rho_E(\frac{1}{6}g_E)}{(\frac{3}{5}\rho_M)}$$

YOUR NOTES

$$r_E \qquad \rho_M g_E \qquad (\tfrac{}{5}\rho_E)g_E$$

$$\frac{r_M}{r_E} = \frac{5}{3} \times \frac{1}{6} = \frac{5}{18} = \mathbf{0.28} \text{ (2 s.f.)}$$

YOUR NOTES

Exam Tip

There is a big difference between *g* and *G* (sometimes referred to as 'little g' and 'big G' respectively), *g* is the gravitational field strength and *G* is Newton's gravitational constant. Make sure not to use these interchangeably! Remember the equation density ρ = mass m ÷ volume V, which may come in handy with some calculations

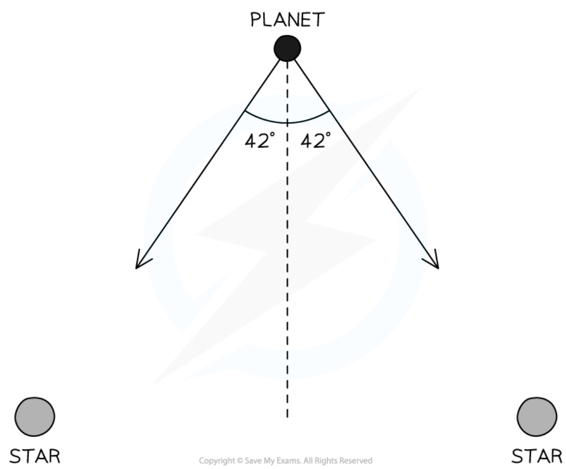
Resultant Gravitational Field Strength

YOUR NOTES

- In a similar way to other vectors, such as force or velocity, the gravitational field strength due to two bodies can be determined
  - This is because gravitational field strength is a vector, meaning it has both a magnitude and direction
- The resultant gravitational field strength is, therefore, the vector sum of the gravitational field strength due to each body

Worked Example

A planet is equidistant from two stars in a binary system. Each star has a mass of 5.0×10^{30} kg and the planet is at a distance of 3.0×10^{12} m from each star. Calculate the magnitude of the resultant gravitational field strength at the position of the planet.



Step 1: List the known quantities

- Mass of one star, $M = 5.0 \times 10^{30}$ kg
- Distance between one star and the planet, $r = 3.0 \times 10^{12}$ m
- Angle between the gravitational field strength and the planet, $\theta = 42°$

Step 2: Write out the equation for gravitational field strength

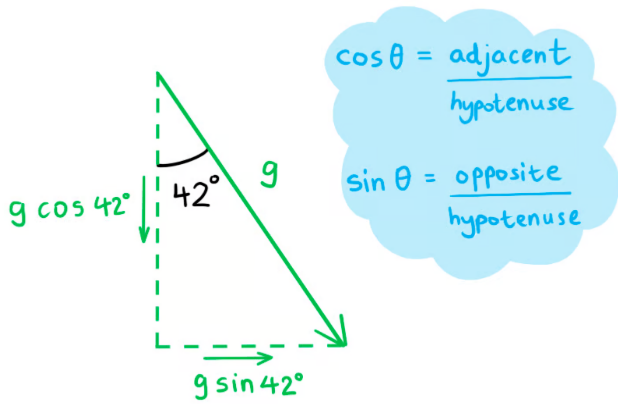
$$g = \frac{GM}{r^2}$$

Step 3: Calculate the gravitational field strength due to one star

$$g = \frac{(6.67 \times 10^{-11}) \times (5.0 \times 10^{30})}{(3.0 \times 10^{12})^2}$$

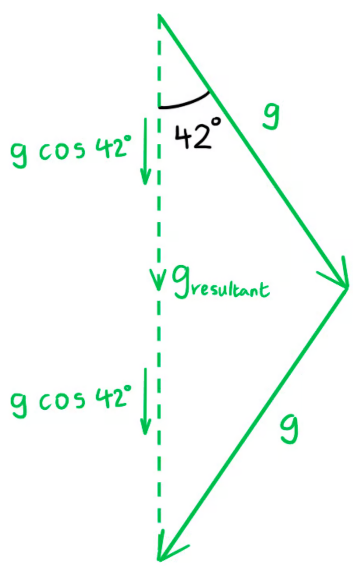
$$g = 3.7 \times 10^{-5} \text{ N kg}^{-1}$$

Step 4: Resolve the vectors vertically



- The vertical component of each vector is $g \cos 42°$

Step 5: Use vector addition to determine the resultant gravitational field strength



$$g_{resultant} = g\cos 42° + g\cos 42° = 2g\cos 42°$$

$$g_{resultant} = 2 \times (3.7 \times 10^{-5}) \times \cos 42°$$

$$g_{resultant} = 5.5 \times 10^{-5} \text{ N kg}^{-1}$$

Exam Tip

Don't worry, for calculation questions involving resultant gravitational field strength - only two bodies along a straight line will be tested!

YOUR NOTES