

3.4 Further Trigonometry

Question Paper

Course	DPIB Maths
Section	3. Geometry & Trigonometry
Topic	3.4 Further Trigonometry
Difficulty	Medium

Time allowed: 90
Score: /68
Percentage: /100

Question 1

Complete the table.

Degrees	Radians	sin	cos	tan
	$\frac{\pi}{6}$		$\frac{\sqrt{3}}{2}$	
45°			$\frac{1}{\sqrt{2}}$	
60°	$\frac{\pi}{3}$			
	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$		
270°				

[5 marks]

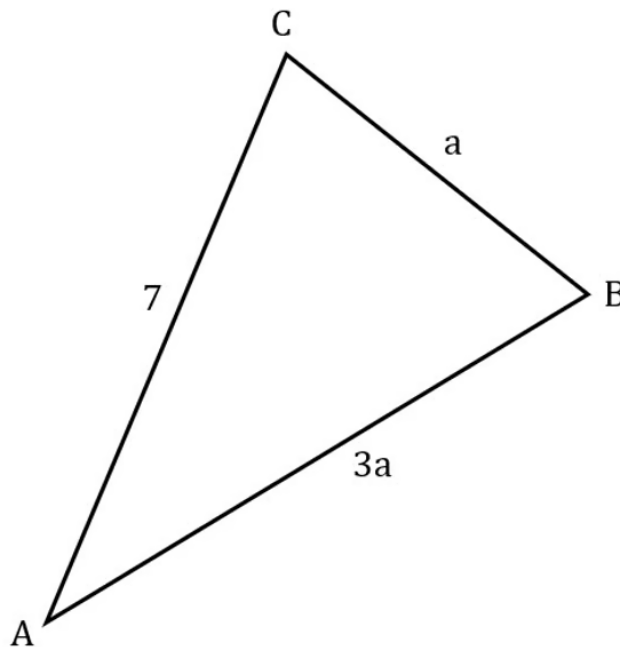
Question 2

 Given that $\sin \theta = \frac{3}{5}$, where $\frac{\pi}{2} < \theta < \pi$, find the possible values of $\cos \theta$ and $\tan \theta$.

[3 marks]

Question 3

The following triangle shows triangle ABC, with $AB = 3a$, $BC = a$ and $AC = 7$.

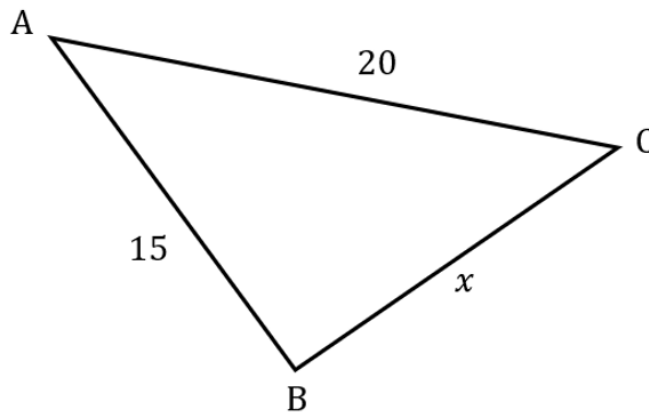


Given that $\cos \widehat{ABC} = \frac{1}{2}$, find the area of the triangle. Give your answer in the form $\frac{p\sqrt{3}}{r}$ where $p, q \in \mathbb{R}$.

[7 marks]

Question 4a

The following triangle shows triangle ABC, with $AB = 15$, $AC = 20$, $BC = x$.



(a) Given that $\cos \widehat{BAC} = \frac{2}{3}$, find the value of $\sin \widehat{BAC}$.

[3 marks]

Question 4b

(b) Find the exact area of triangle ABC.

[3 marks]

Question 4c

(c) By finding the value of x , show that triangle ABC is isosceles.

[3 marks]

Question 5

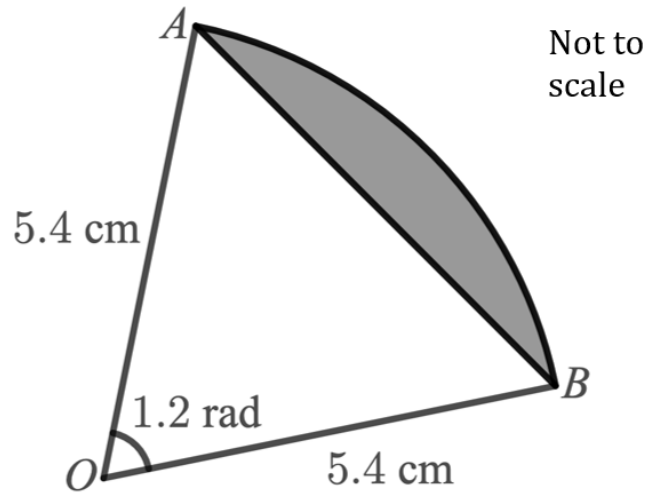
A sector of a circle, OPQ , is such that it has radius 3.4 cm and the angle at its centre, O , is $\frac{3\pi}{4}$ radians.

- (i) Find the length of the arc PQ .
- (ii) Find the area of the sector OPQ .

[4 marks]

Question 6a

The diagram below shows the sector of a circle OAB .



- (a) (i) Find the area of the sector OAB , giving your answer to 3 significant figures.
- (ii) Find the area of the triangle OAB , giving your answer to 3 significant figures.
- (iii) Find the area of the shaded segment, giving your answer to 3 significant figures.

[5 marks]

Question 6b

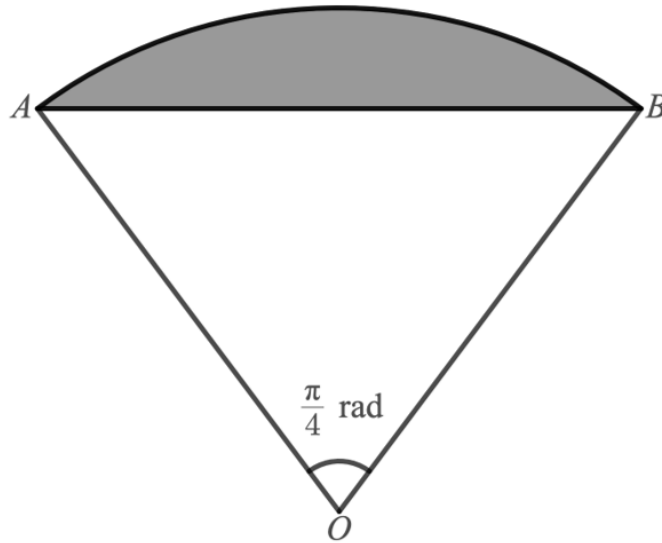
(b) (i) Find the length of the arc AB .

(ii) Find the perimeter of the sector OAB .

[3 marks]

Question 7

The canopy of a parachute and the outermost connecting cords form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O .



Not to scale

The area of the sector OAB is $\frac{81\pi}{200}$ m².

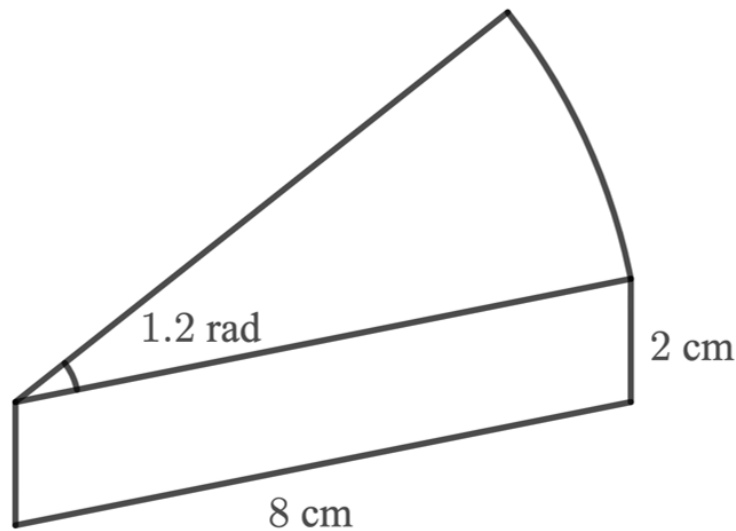
Find the length of one of the connecting cords on the parachute.

[3 marks]

Question 8

A plastic puzzle piece is in the form of a prism with a cross-section that is the sector of a circle, as shown in the diagram below. The radius of the sector is 8 cm, and the angle at the centre is 1.2 radians.

The height of the puzzle piece is 2 cm.



Not to scale

- (i) Work out the area of the cross-section.
- (ii) Hence, or otherwise, work out the volume of the puzzle piece.

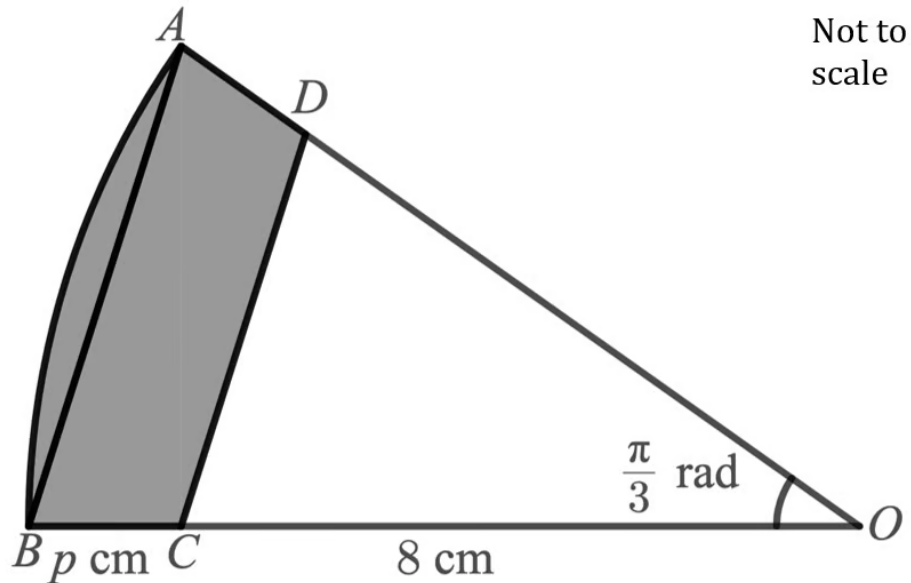
[3 marks]

Question 9a

The circle sector OAB is shown in the diagram below.

The angle at the centre is $\frac{\pi}{3}$ radians, and the line segments OC and BC have lengths of 8 cm and p cm respectively.

Additionally, CD is parallel to AB , so that $AD = BC$ and $OD = OC$.



(a) Show that the area of the sector OAB is $\frac{\pi}{6}(p + 8)^2 \text{ cm}^2$.

[2 marks]

Question 9b

(b) Show that the area of the triangle OCD is $16\sqrt{3} \text{ cm}^2$.

[2 marks]

Question 9c

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right) \text{ cm}^2$, find the value of p .

[4 marks]**Question 10**

Solve the equation $2 \sin x = \frac{1}{\sin x}$ for $0^\circ \leq x \leq 360^\circ$.

[5 marks]

Question 11

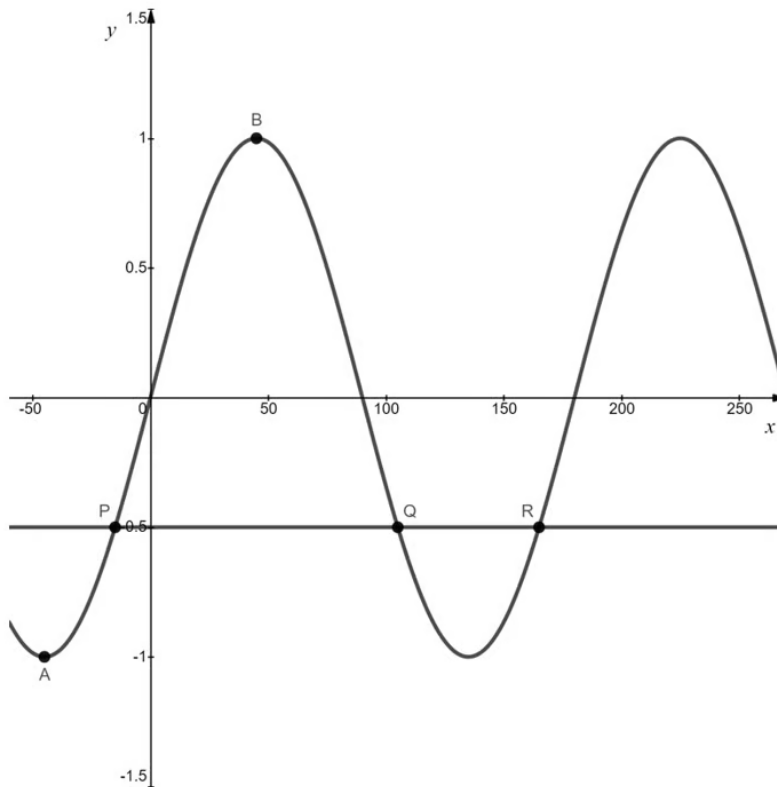
A right-angled triangle has hypotenuse 8cm. One of its other sides is 5cm.

Find exact values for $\sin \theta$, $\cos \theta$ and $\tan \theta$, where θ is the smallest angle in the triangle.

[6 marks]

Question 12a

The graph below shows the curve with equation $y = \sin 2x$ in the interval $-60^\circ \leq x \leq 270^\circ$.



- (a) Point A has coordinates $(-45^\circ, -1)$ and is the minimum point closest to the origin. Point B is the maximum point closest to the origin. State the coordinates of B .

[1 mark]

Question 12b

(b) A straight line with equation $y = -\frac{1}{2}$ meets the graph of $y = \sin 2x$ at the three points P , Q and R , as shown in the diagram.

Given that point P has coordinates $(-15^\circ, -\frac{1}{2})$, use graph symmetries to determine the coordinates of Q and R .

[2 marks]

Question 13

(i) Sketch the graph of $y = \cos(\theta + 30^\circ)$ in the interval $-180^\circ \leq \theta \leq 360^\circ$.

(ii) Write down all the values where $\cos(\theta + 30^\circ) = 0$ in the given interval.

[4 marks]