

# 5.1 Differentiation

## Question Paper

Course	DPIB Maths
Section	5. Calculus
Topic	5.1 Differentiation
Difficulty	Medium

**Time allowed:** 110  
**Score:** /86  
**Percentage:** /100

**Question 1a**

The equation of a curve is  $y = \frac{3}{2}x^2 - 15x + 2$ .

(a) Find  $\frac{dy}{dx}$ .

[2 marks]

**Question 1b**

The gradient of the tangent to the curve at point A is  $-3$ .

(b) Find

- (i) the coordinates of A
- (ii) the equation of the tangent to the curve at point A.  
Give your answer in the form  $y = mx + c$ .

[4 marks]

**Question 2a**

Consider the function  $f(x) = 3x^7 - 12x$ .

(a) Find  $f'(x)$ .

[1 mark]

**Question 2b**

(b) Find the gradient of the graph of  $f$  at  $x = 0$ .

[2 marks]

**Question 2c**

(c) Find the coordinates of the points at which the normal to the graph of  $f$  has a gradient of 4.

[3 marks]

**Question 3a**

The equation of a curve is  $y = 4 - \frac{4}{x}$ .

- (a) Find the equation of the tangent to the curve at  $x = 2$ .  
Give your answer in the form  $y = mx + c$ .

[3 marks]

**Question 3b**

- (b) Find the coordinates of the points on the curve where the gradient is 16.

[3 marks]

**Question 4a**

Consider the function  $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5}$ ,  $x \neq 0$ .

(a) Calculate

(i)  $f(2)$

(ii)  $f'(2)$ .

[3 marks]

**Question 4b**

A line,  $l$ , is tangent to the graph of  $y = f(x)$  at the point  $x = 2$ .

(b) Find the equation of  $l$ . Give your answer in the form  $y = mx + c$ .

[3 marks]

**Question 4c**

The graph of  $y = f(x)$  and  $l$  have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2 marks]

**Question 5a**

Consider the function  $f(x) = x^2 - bx + c$ .

(a) Find  $f'(x)$ .

[1 mark]

**Question 5b**

The equation of the tangent line to the graph  $y = f(x)$  at  $x = 2$  is  $y = x - 1$ .

(b) Calculate the value of  $b$ .

[2 marks]

**Question 5c**

(c) Calculate the value of  $c$  and write down the function  $f(x)$ .

[3 marks]

**Question 6a**

The curve with equation  $y = ax^2 + bx + c$  has a gradient of  $-7$  at the point  $(-1, 13)$ , and a gradient of  $-3$  at the point  $(1, 3)$ .

(a) By considering  $\frac{dy}{dx}$  show that  $2a + b = -3$  and  $-2a + b = -7$ .

[2 marks]

**Question 6b**

(b) Hence find the values of  $a$  and  $b$ .

[1 mark]

**Question 6c**

(c) By considering a point that you know to be on the curve, find the value of  $c$ .

[2 marks]

**Question 7a**

The curve  $C$  has equation  $y = 3x^2 - 6 + \frac{4}{x}$ . The point  $P(1, 1)$  lies on  $C$ .

(a) Find an expression for  $\frac{dy}{dx}$ .

[2 marks]

**Question 7b**

(b) Show that an equation of the normal to  $C$  at point  $P$  is  $x + 2y = 3$ .

[3 marks]

**Question 7c**

This normal cut the  $x$ -axis at the point  $Q$ .

(c) Find the length of  $PQ$ , giving your answer as an exact value.

[2 marks]



**Question 8**

Find the values of  $x$  for which  $f(x) = -9x^2 + 5x - 3$  is an increasing function.

[3 marks]

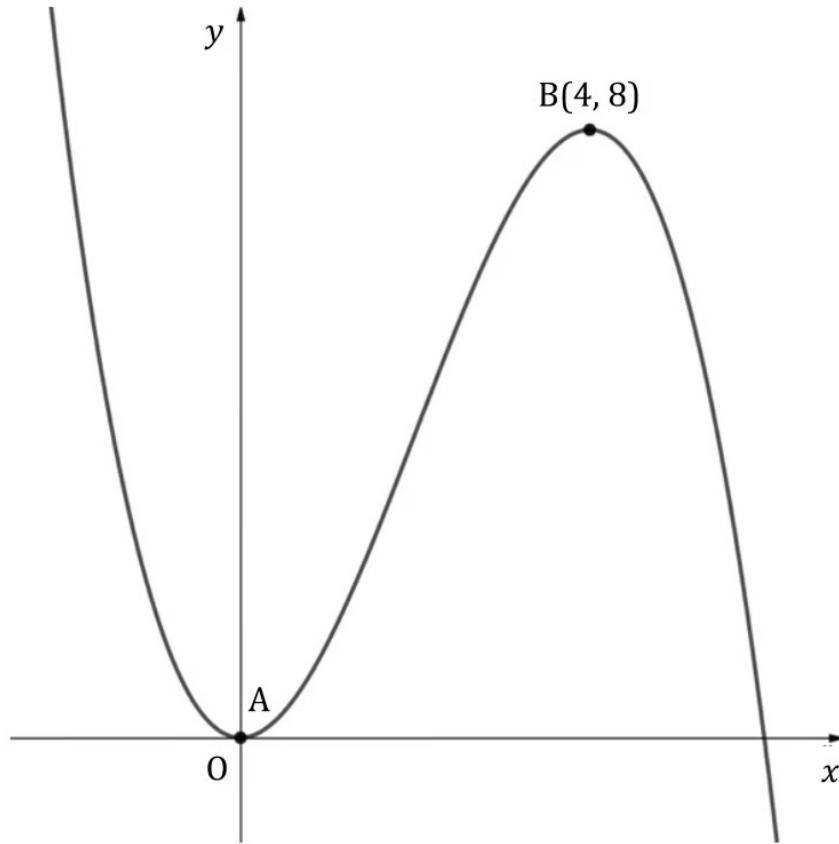
**Question 9**

Show that the function  $f(x) = x^3 - 3x^2 + 6x - 7$  is increasing for all  $x \in \mathbb{R}$ .

[3 marks]

**Question 10a**

The graph of the cubic function  $y = f(x)$  is shown below. Point  $A$ , a local minimum, is located at the origin and point  $B$ , a local maximum, sits at the point  $(4, 8)$ .



(a) State the equations of the horizontal tangent to the curve.

[2 marks]

**Question 10b**

(b) Write down the value of  $x$  where the point of inflection is located.

[1 mark]

**Question 10c**

(c) Find the intervals where  $f$  is decreasing.

[2 marks]

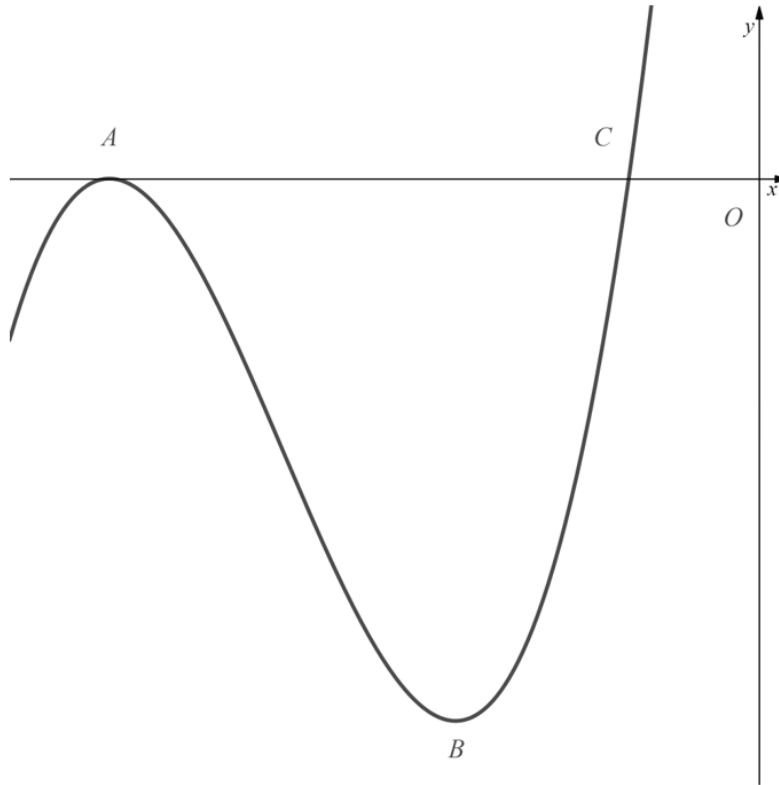
**Question 10d**

(d) Sketch the graph of  $f'(x)$ , labelling clearly any intercepts and axis of symmetry.

[3 marks]

**Question 11a**

The diagram below shows part of the curve with equation  $y = x^3 + 11x^2 + 35x + 25$ . The curve touches the  $x$ -axis at  $A$  and cuts the  $x$ -axis at  $C$ . The points  $A$  and  $B$  are stationary points on the curve.



(a) Using calculus, and showing all your working, find the coordinates of  $A$  and  $B$ .

[5 marks]

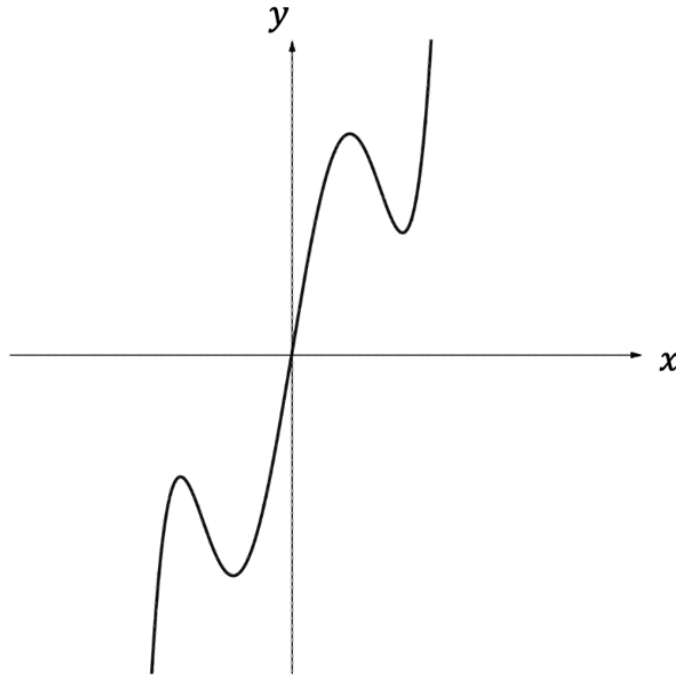
**Question 11b**

(b) Show that  $(-1, 0)$  is a point on the curve and explain why those must be the coordinates of point  $C$ .

[2 marks]

**Question 12a**

The equation of the curve  $C$  is  $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$ . A section of the curve  $C$  is shown on the diagram below.



(a) Find  $\frac{dy}{dx}$ .

[2 marks]

**Question 12b**

There are two points, R and S, along the curve  $C$  at which the gradient of the tangent to the curve  $C$  is equal to 10.

(b) Calculate the  $x$ -coordinates of points R and S.

[4 marks]

**Question 13a**

- (a) Find the  $x$ -coordinates of the stationary points on the graph with equation  
 $y = x^3 - 6x^2 + 9x - 1$ .

[4 marks]

**Question 13b**

- (b) Find the nature of the stationary points found in part (a).

[3 marks]

**Question 13c**

- (c) Determine the  $x$ -coordinate of the point of inflection on the graph with equation  $y = x^3 - 6x^2 + 9x - 1$ .

[3 marks]

**Question 13d**

- (d) Explain why, in this case, the point of inflection is not a stationary point.

[1 mark]



**Question 14**

The graph of a continuous function has the following properties:

The function is concave in the interval  $(-\infty, a)$ .

The function is convex in the interval  $(a, \infty)$ .

The graph of the function intercepts the  $x$ -axis at the points  $(b, 0)$ ,  $(c, 0)$  and  $(d, 0)$ , where  $b, c$  and  $d$  are such that  $d > c > b > 0$ .

The  $x$ -coordinates of the turning points of the function are  $e$  and  $f$ , which are such that  $f > e$ .

The graph of the function intercepts the  $y$ -axis at  $(0, g)$

Given that the value of the function is positive when  $x = a$ , sketch a graph of the function. Be sure to label the  $x$ -axis with the  $x$ -coordinates of the stationary points and the point of inflection, and also to label the points where the graph crosses the coordinate axes.

[4 marks]