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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

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Tuesday 12 November 2013 (morning)

1 hour 30 minutes

Examination code

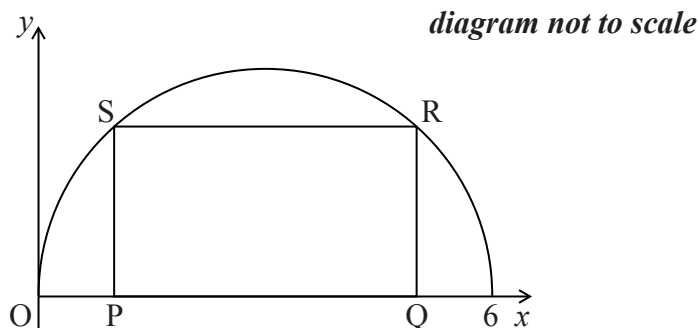| 8	8	1	3	–	7	3	0	2

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $A = \begin{pmatrix} 0 & 5 & 4 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix}$, and $B = \begin{pmatrix} 11 \\ 7 \\ 10 \end{pmatrix}$.

(a) Write down A^{-1}. *[2]*

(b) Hence or otherwise, solve the equation $AX = B$. *[3]*

2.   *[Maximum mark: 6]*

Let $f(x) = (x-1)(x-4)$.

(a) Find the x-intercepts of the graph of f.   *[3]*

(b) The region enclosed by the graph of f and the x-axis is rotated $360°$ about the x-axis. Find the volume of the solid formed.   *[3]*

3. *[Maximum mark: 6]*

Let $f(x) = \sqrt[3]{x^4} - \dfrac{1}{2}$.

(a) Find $f'(x)$. *[2]*

(b) Find $\int f(x)\mathrm{d}x$. *[4]*

4.   *[Maximum mark: 6]*

Two events A and B are such that $P(A) = 0.2$ and $P(A \cup B) = 0.5$.

(a)   Given that A and B are mutually exclusive, find $P(B)$.   *[2]*

(b)   Given that A and B are independent, find $P(B)$.   *[4]*

Turn over

5.   *[Maximum mark: 8]*

A particle moves along a straight line such that its velocity, $v\,\text{ms}^{-1}$, is given by $v(t) = 10t\text{e}^{-1.7t}$, for $t \geq 0$.

(a)   On the grid below, sketch the graph of v, for $0 \leq t \leq 4$.   *[3]*

(b)   Find the distance travelled by the particle in the first three seconds.   *[2]*

(c)   Find the velocity of the particle when its acceleration is zero.   *[3]*

6.   *[Maximum mark: 7]*

The time taken for a student to complete a task is normally distributed with a mean of 20 minutes and a standard deviation of 1.25 minutes.

(a)   A student is selected at random. Find the probability that the student completes the task in less than 21.8 minutes.   *[2]*

(b)   The probability that a student takes between k and 21.8 minutes is 0.3. Find the value of k.   *[5]*

7. *[Maximum mark: 7]*

Consider the graph of the semicircle given by $f(x) = \sqrt{6x - x^2}$, for $0 \le x \le 6$. A rectangle PQRS is drawn with upper vertices R and S on the graph of f, and PQ on the x-axis, as shown in the following diagram.

diagram not to scale

(a)   Let $OP = x$.

   (i)   Find PQ, giving your answer in terms of x.

   (ii)   Hence, write down an expression for the area of the rectangle, giving your answer in terms of x. *[3]*

(b)   (i)   Find the rate of change of area when $x = 2$.

   (ii)   The area is decreasing for $a < x < b$. Find the value of a and of b. *[4]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider a circle with centre O and radius 7 cm. Triangle ABC is drawn such that its vertices are on the circumference of the circle.

diagram not to scale
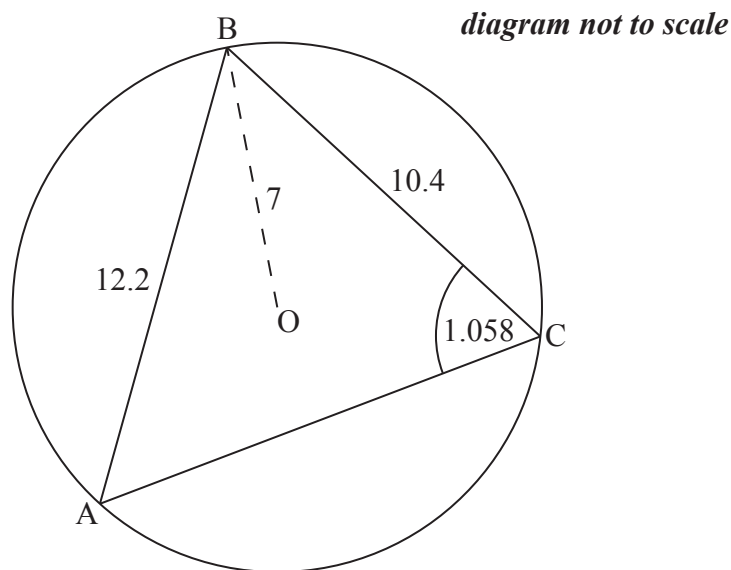
$AB = 12.2 \, \text{cm}$, $BC = 10.4 \, \text{cm}$ and $A\hat{C}B = 1.058$ radians.

(a) Find $B\hat{A}C$. *[3]*

(b) Find AC. *[5]*

(c) Hence or otherwise, find the length of arc ABC. *[6]*

Turn over

*Do **NOT** write solutions on this page.*

9.   *[Maximum mark: 17]*

Consider the lines L_1 and L_2 with equations $L_1 : r = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and $L_2 : r = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P.

(a)   Find the coordinates of P.   *[6]*

(b)   Show that the lines are perpendicular.   *[5]*

(c)   The point $Q(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2.
       Find the coordinates of R.   *[6]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

Samantha goes to school five days a week. When it rains, the probability that she goes to school by bus is 0.5. When it does not rain, the probability that she goes to school by bus is 0.3. The probability that it rains on any given day is 0.2.

(a) On a randomly selected school day, find the probability that Samantha goes to school by bus. *[4]*

(b) Given that Samantha went to school by bus on Monday, find the probability that it was raining. *[3]*

(c) In a randomly chosen school week, find the probability that Samantha goes to school by bus on exactly three days. *[2]*

(d) After *n* school days, the probability that Samantha goes to school by bus at least once is greater than 0.95. Find the smallest value of *n*. *[5]*

Please **do not** write on this page.

Answers written on this page
will not be marked.