22137305

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 9 May 2013 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

2	2	1	3	–	7	3	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.

(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(1)$. *[3 marks]*

Turn over

2. *[Maximum mark: 6]*

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 1 & q \end{pmatrix}$, $C = \begin{pmatrix} p & -1 \\ 6 & 3 \end{pmatrix}$, so that $AB = C$.

(a)   Find the value of p.   *[3 marks]*

(b)   Find the value of q.   *[3 marks]*

0416

3.   *[Maximum mark: 7]*

Let $\log_3 p = 6$ and $\log_3 q = 7$.

(a)   Find $\log_3 p^2$.   *[2 marks]*

(b)   Find $\log_3\left(\dfrac{p}{q}\right)$.   *[2 marks]*

(c)   Find $\log_3(9p)$.   *[3 marks]*
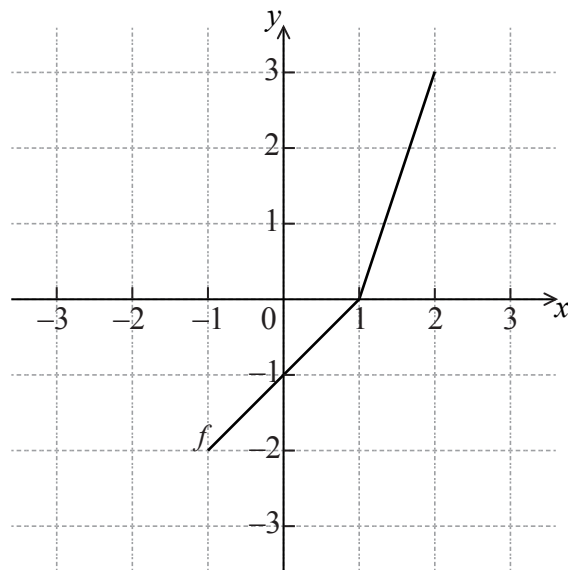
Turn over

4. *[Maximum mark: 6]*

The diagram below shows the graph of a function f, for $-1 \le x \le 2$.

(a) Write down the value of

    (i) $f(2)$;

    (ii) $f^{-1}(-1)$.   *[3 marks]*

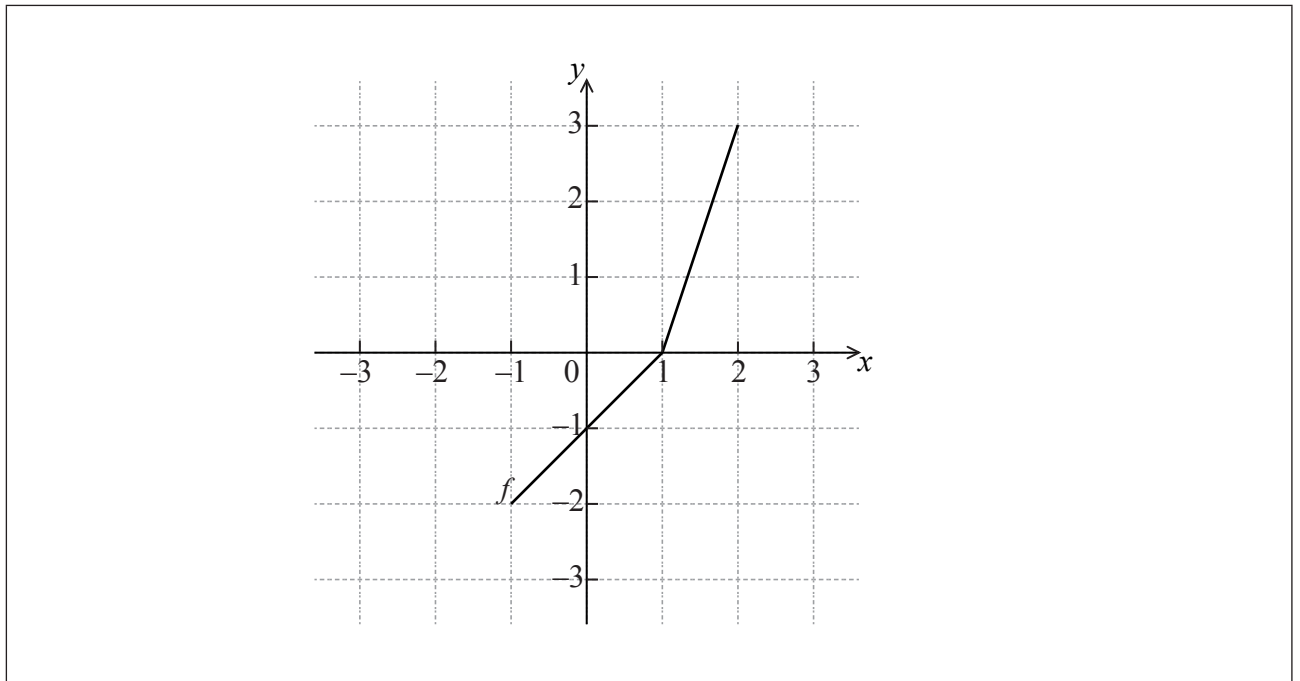*(This question continues on the following page)*

(Question 4 continued)

(b)   Sketch the graph of f^{-1} on the grid below.   *[3 marks]*

Turn over

5.   *[Maximum mark: 6]*

The diagram below shows part of the graph of a function f.
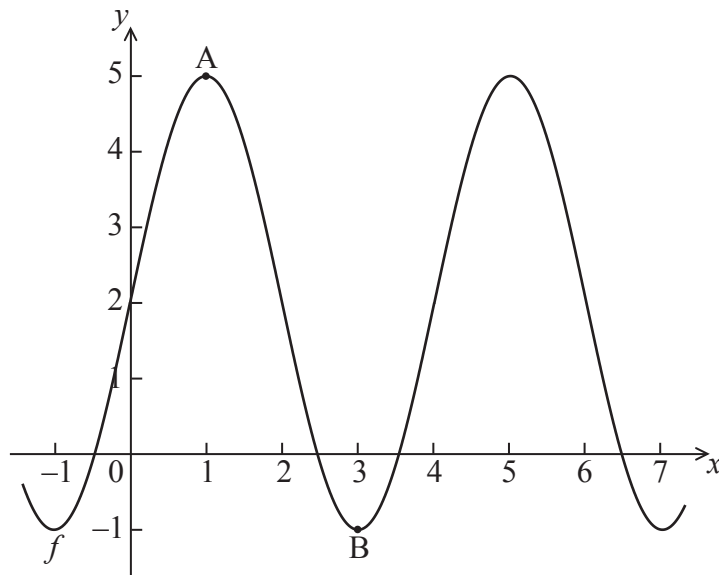
The graph has a maximum at $A(1, 5)$ and a minimum at $B(3, -1)$.

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

(a) p;   *[2 marks]*

(b) q;   *[2 marks]*

(c) r.   *[2 marks]*

6. *[Maximum mark: 7]*

A rocket moving in a straight line has velocity v km s^{-1} and displacement s km at time t seconds. The velocity v is given by $v(t) = 6e^{2t} + t$. When $t = 0$, $s = 10$. Find an expression for the displacement of the rocket in terms of t.
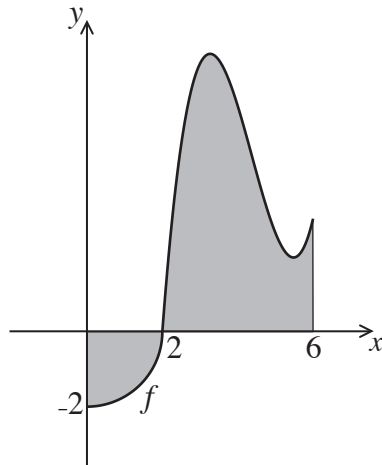
Turn over

7. *[Maximum mark: 7]*

The following is the graph of a function f, for $0 \le x \le 6$.

The first part of the graph is a quarter circle of radius 2 with centre at the origin.

(a) Find $\int_0^2 f(x)\,dx$. *[4 marks]*

(b) The shaded region is enclosed by the graph of f, the x-axis, the y-axis and the line $x = 6$. The area of this region is 3π.

Find $\int_2^6 f(x)\,dx$. *[3 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

A running club organizes a race to select girls to represent the club in a competition. The times taken by the group of girls to complete the race are shown in the table below.

Time t minutes	$10 \leq t < 12$	$12 \leq t < 14$	$14 \leq t < 20$	$20 \leq t < 26$	$26 \leq t < 28$	$28 \leq t < 30$
Frequency	50	20	p	40	20	20
Cumulative Frequency	50	70	120	q	180	200

(a) Find the value of p and of q. *[4 marks]*

(b) A girl is chosen at random.

    (i) Find the probability that the time she takes is less than 14 minutes.

    (ii) Find the probability that the time she takes is at least 26 minutes. *[3 marks]*

A girl is selected for the competition if she takes less than x minutes to complete the race.

(c) Given that 40 % of the girls are not selected,

    (i) find the number of girls who are not selected;

    (ii) find x. *[4 marks]*
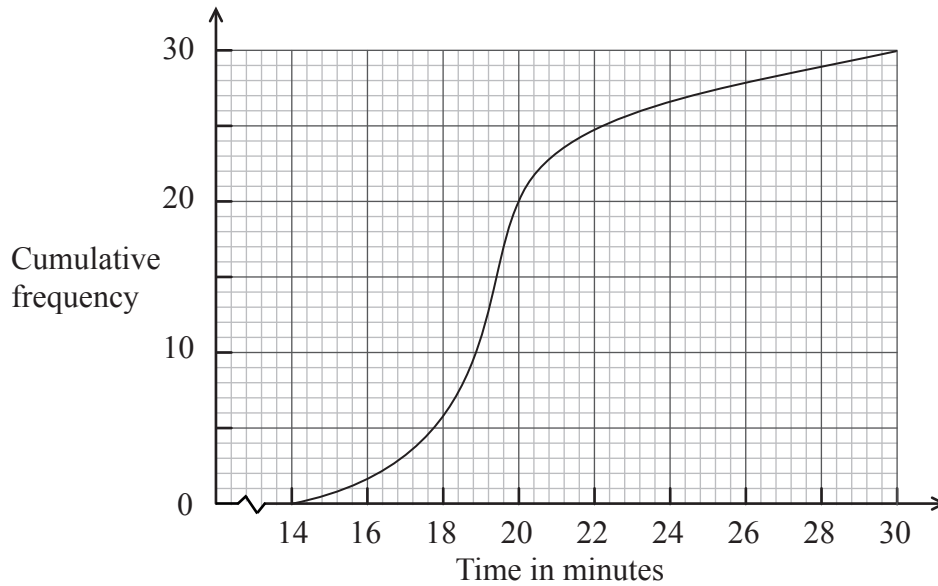
*Do **NOT** write solutions on this page.*

(Question 8 continued)

Girls who are not selected, but took less than 25 minutes to complete the race, are allowed another chance to be selected. The new times taken by these girls are shown in the cumulative frequency diagram below.

(d)   (i)   Write down the number of girls who were allowed another chance.

(ii)   Find the percentage of the **whole** group who were selected.   *[4 marks]*

Turn over

*Do **NOT** write solutions on this page.*

9.   *[Maximum mark: 16]*

Let $f(x) = \sin x + \dfrac{1}{2}x^2 - 2x$, for $0 \le x \le \pi$.

(a)   Find $f'(x)$.   *[3 marks]*

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g.

(b)   Find $g(4)$.   *[3 marks]*

The function g can be expressed in the form $g(x) = a(x-h)^2 + 3$.

(c)   (i)   Write down the value of h.

     (ii)   Find the value of a.   *[4 marks]*

(d)   Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g.   *[6 marks]*
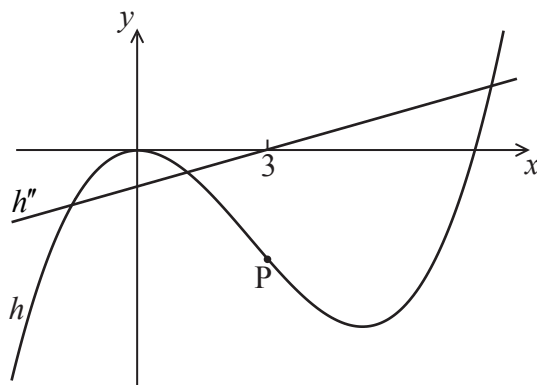
*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	−14	−18
$f'(x)$	1	1
$g'(x)$	−5	−3
$h''(x)$	−6	0

(a) Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. *[3 marks]*

The following diagram shows parts of the graphs of h and h''.

There is a point of inflexion on the graph of h at P, when $x = 3$.

(b) Explain why P is a point of inflexion. *[2 marks]*

Given that $h(x) = f(x) \times g(x)$,

(c) find the *y*-coordinate of P; *[2 marks]*

(d) find the equation of the normal to the graph of h at P. *[7 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.