22107304

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 6 May 2010 (morning)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 2 & 4 & -3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Solve $\boldsymbol{AX} = \boldsymbol{B}$. *[3 marks]*

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2. *[Maximum mark: 6]*

Consider the arithmetic sequence $3, 9, 15, \ldots, 1353$.

(a) Write down the common difference. *[1 mark]*

(b) Find the number of terms in the sequence. *[3 marks]*

(c) Find the sum of the sequence. *[2 marks]*

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3. *[Maximum mark: 7]*

Let $f(x) = x\cos x$, for $0 \le x \le 6$.

(a) Find $f'(x)$. *[3 marks]*

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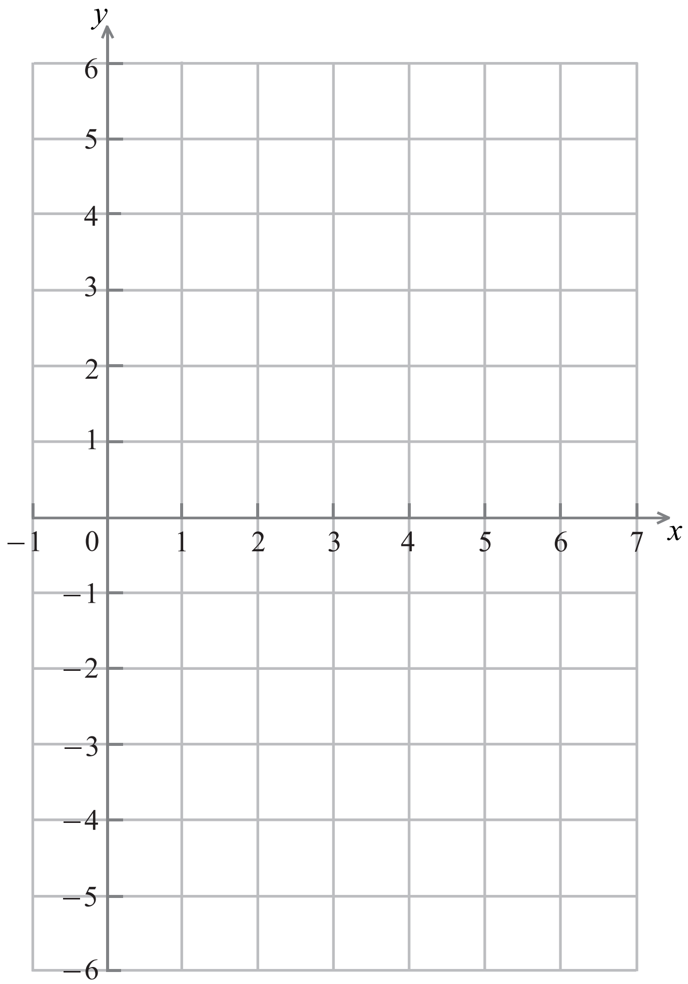
(b) On the grid below, sketch the graph of $y = f'(x)$. *[4 marks]*



4. *[Maximum mark: 6]*

The following frequency distribution of marks has mean 4.5.

Mark	1	2	3	4	5	6	7
Frequency	2	4	6	9	x	9	4

(a) Find the value of x. *[4 marks]*

(b) Write down the standard deviation. *[2 marks]*

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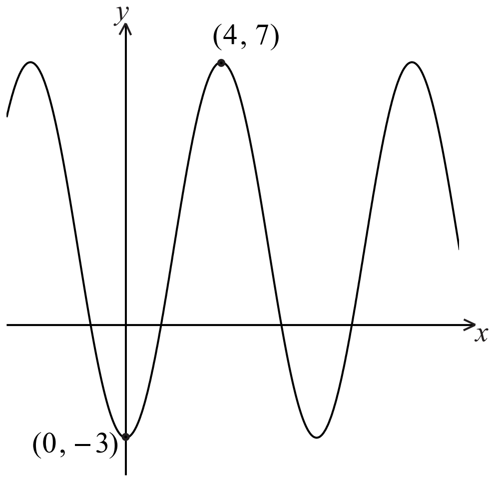
5. *[Maximum mark: 7]*

The graph of $y = p\cos qx + r$, for $-5 \le x \le 14$, is shown below.



There is a minimum point at $(0, -3)$ and a maximum point at $(4, 7)$.

(a) Find the value of

(i) p;

(ii) q;

(iii) r. *[6 marks]*

(b) The equation $y = k$ has exactly **two** solutions. Write down the value of k. *[1 mark]*

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6. *[Maximum mark: 7]*

The acceleration, $a\,\text{m}\,\text{s}^{-2}$, of a particle at time t seconds is given by

$$a = \frac{1}{t} + 3\sin 2t\text{, for } t \geq 1.$$

The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

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7. *[Maximum mark: 7]*

Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games. *[2 marks]*

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games. *[2 marks]*

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15. *[3 marks]*

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*Do **NOT** write on this page.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The diagram below shows a quadrilateral ABCD with obtuse angles $\text{A}\hat{\text{B}}\text{C}$ and $\text{A}\hat{\text{D}}\text{C}$.

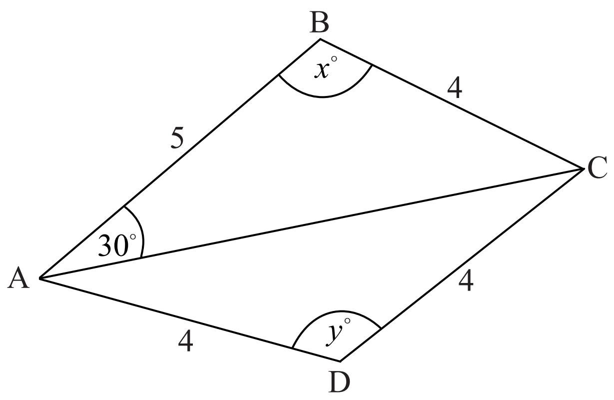

diagram not to scale

$\text{AB} = 5 \text{ cm}$, $\text{BC} = 4 \text{ cm}$, $\text{CD} = 4 \text{ cm}$, $\text{AD} = 4 \text{ cm}$, $\text{B}\hat{\text{A}}\text{C} = 30^\circ$, $\text{A}\hat{\text{B}}\text{C} = x^\circ$, $\text{A}\hat{\text{D}}\text{C} = y^\circ$.

(a) Use the cosine rule to show that $\text{AC} = \sqrt{41 - 40\cos x}$. *[1 mark]*

(b) Use the sine rule in triangle ABC to find another expression for AC. *[2 marks]*

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC. *[6 marks]*

(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD. *[5 marks]*

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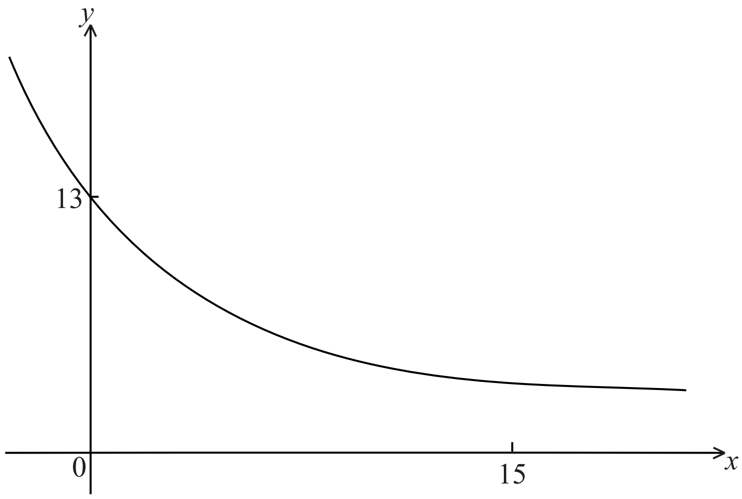
9. *[Maximum mark: 16]*

Let $f(x) = A\mathrm{e}^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at $(0, 13)$.

(a) Show that $A = 10$. *[2 marks]*

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k. *[3 marks]*

(c) (i) Using your value of k, find $f'(x)$.

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f. *[5 marks]*

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of f and g. *[6 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 15]*

The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg. *[2 marks]*

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights. *[4 marks]*

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament. *[5 marks]*

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman. *[4 marks]*