22107303

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 5 May 2010 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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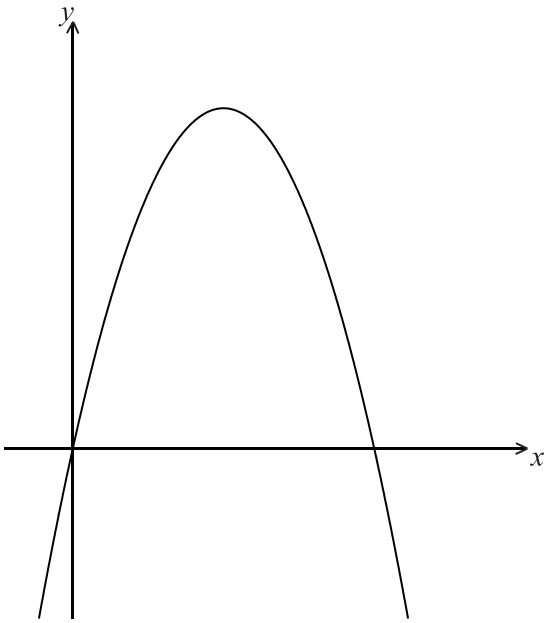
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 7]*

Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.



(a) Find the x-intercepts of the graph. *[4 marks]*

(b) (i) Write down the equation of the axis of symmetry.

(ii) Find the y-coordinate of the vertex. *[3 marks]*

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2. *[Maximum mark: 6]*

Let $\boldsymbol{W} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ and $\boldsymbol{P} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find $\boldsymbol{WP}$. *[3 marks]*

(b) Given that $2\boldsymbol{WP} + \boldsymbol{S} = \begin{pmatrix} 26 \\ 12 \\ 10 \end{pmatrix}$, find $\boldsymbol{S}$. *[3 marks]*

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3. *[Maximum mark: 6]*

(a) Expand $(2+x)^4$ and simplify your result. *[3 marks]*

(b) Hence, find the term in x^2 in $(2+x)^4\left(1+\frac{1}{x^2}\right)$. *[3 marks]*

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4. *[Maximum mark: 7]*

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x-axis.

(a) Write down the value of $\tan\theta$. *[1 mark]*

(b) Find the value of

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$. *[6 marks]*

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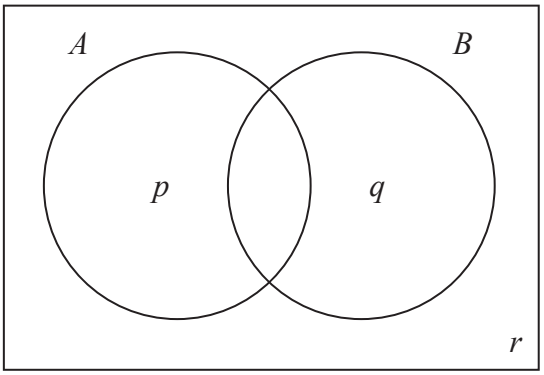
5. *[Maximum mark: 6]*

Consider the events A and B, where $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.7$ and $\mathrm{P}(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.



(a) Write down the value of

(i) p;

(ii) q;

(iii) r. *[3 marks]*

(b) Find the value of $\mathrm{P}(A \mid B')$. *[2 marks]*

(c) Hence, or otherwise, show that the events A and B are **not** independent. *[1 mark]*

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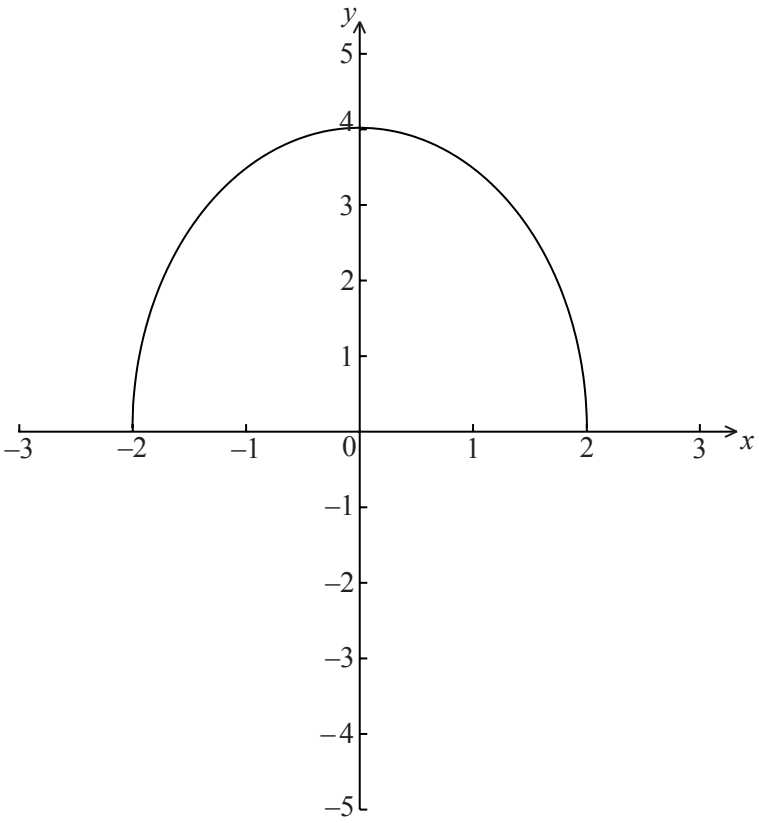
6. *[Maximum mark: 6]*

The graph of $f(x) = \sqrt{16 - 4x^2}$, for $-2 \le x \le 2$, is shown below.



The region enclosed by the curve of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed.

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7. *[Maximum mark: 7]*

Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$. *[2 marks]*

(b) Write down the range of f^{-1}. *[1 mark]*

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer. *[4 marks]*

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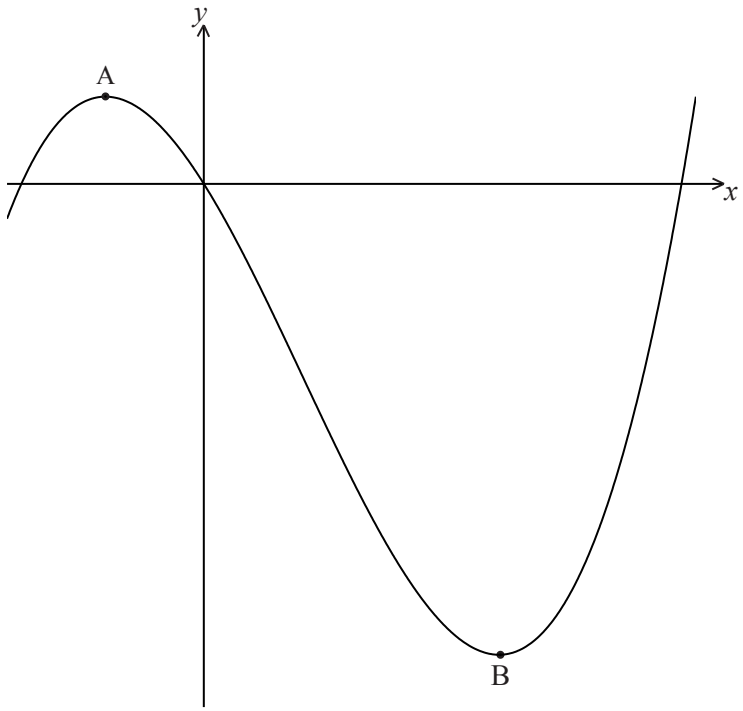
SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at $\mathrm{B}(3, -9)$.

(a) Find the coordinates of A. *[8 marks]*

(b) Write down the coordinates of

(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$. *[6 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

(a) Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. *[5 marks]*

(b) Find $f''(x)$. *[3 marks]*

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2} - 0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.1$
$f'(x)$	–1.01	p	–1.01
$f''(x)$	0.203	q	-0.203

(c) Find the value of p and of q. *[3 marks]*

(d) Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$. *[2 marks]*

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10. *[Maximum mark: 18]*

The line L_1 is represented by the vector equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point $\mathrm{B}(-8, -5, 25)$.

(a) Write down a vector equation for L_2 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

A third line L_3 is perpendicular to L_1 and is represented by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

(b) Show that $k = -2$. *[5 marks]*

The lines L_1 and L_3 intersect at the point A.

(c) Find the coordinates of A. *[6 marks]*

The lines L_2 and L_3 intersect at point C where $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

(d) (i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, find $|\overrightarrow{\mathrm{AC}}|$. *[5 marks]*