88097302

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 6 November 2009 (morning)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

In an arithmetic sequence, $S_{40} = 1900$ and $u_{40} = 106$. Find the value of u_1 and of d.

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2. *[Maximum mark: 6]*

Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

(a) Find $f'(x)$. *[2 marks]*

(b) Find $g'(x)$. *[2 marks]*

(c) Let $h(x) = f(x) \times g(x)$. Find $h'(x)$. *[2 marks]*

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3. *[Maximum mark: 6]*

A multiple choice test consists of ten questions. Each question has five answers. Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a) Find the expected number of questions Jose answers correctly. *[1 mark]*

(b) Find the probability that Jose answers exactly three questions correctly. *[2 marks]*

(c) Find the probability that Jose answers more than three questions correctly. *[3 marks]*

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4. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & -3 & 0 \\ 4 & -2 & 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Let $\boldsymbol{B}$ be a 3×3 matrix. Given that $\boldsymbol{AB} + \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix} = \begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix}$, find $\boldsymbol{B}$. *[4 marks]*

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5. *[Maximum mark: 7]*

Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants. The point $\mathrm{A}(1, 3)$ lies on the curve. The tangent to the curve at A has gradient 8. Find the value of p and of q.

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6. *[Maximum mark: 7]*

Consider the independent events A and B. Given that $\mathrm{P}(B) = 2\,\mathrm{P}(A)$, and $\mathrm{P}(A \cup B) = 0.52$, find $\mathrm{P}(B)$.

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7. *[Maximum mark: 7]*

A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown below.

The fencing used for side AB costs \$11 per metre. The fencing for the other three sides costs \$3 per metre. The farmer creates an enclosure so that the cost is a minimum. Find this minimum cost.

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*Do **NOT** write on this page.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

The following diagram shows a circle with centre O and radius 4 cm.

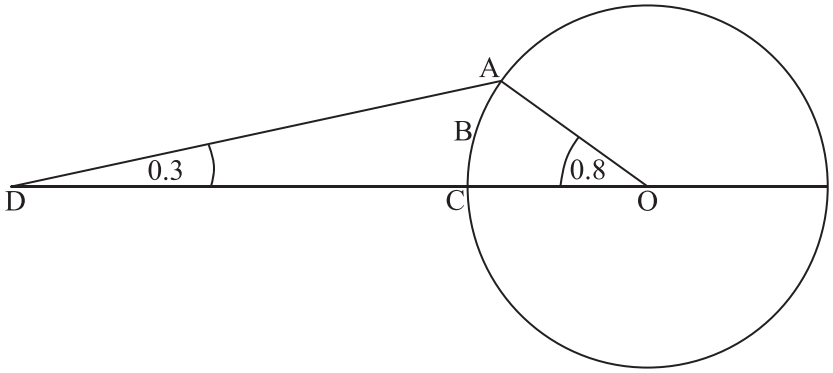

diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC). Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

(a) Find AD. *[3 marks]*

(b) Find OD. *[4 marks]*

(c) Find the area of sector OABC. *[2 marks]*

(d) Find the area of region ABCD. *[4 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.

(a) On the same diagram, sketch the graphs of f and g. *[3 marks]*

(b) Consider the graph of f. Write down

(i) the x-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude. *[4 marks]*

(c) Consider the graph of g. Write down

(i) the two x-intercepts;

(ii) the equation of the axis of symmetry. *[3 marks]*

(d) Let R be the region enclosed by the graphs of f and g. Find the area of R. *[5 marks]*

*Do **NOT** write on this page.*

10. *[Maximum mark: 17]*

Consider the points $\mathrm{P}(2, -1, 5)$ and $\mathrm{Q}(3, -3, 8)$. Let L_1 be the line through P and Q.

(a) Show that $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$. *[1 mark]*

(b) The line L_1 may be represented by $\boldsymbol{r} = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

(i) What information does the vector $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$ give about L_1?

(ii) Write down another vector representation for L_1 using $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$. *[3 marks]*

The point $\mathrm{T}(-1, 5, p)$ lies on L_1.

(c) Find the value of p. *[3 marks]*

The point T also lies on L_2 with equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ q \end{pmatrix}$.

(d) Show that $q = -3$. *[3 marks]*

(e) Let θ be the **obtuse** angle between L_1 and L_2. Calculate the size of θ. *[7 marks]*