International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

November 2009

MATHEMATICS

Standard Level

Paper 1

Deadlines (different to IBIS/Scoris)

Samples to Team Leaders	4 December
Everything to IB Cardiff	11 December

NB. THERE IS NO ACCURACY PENALTY FOR PAPER 1 IN NOVEMBER 2009

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and hints to examiners, using annotations when required, but see point 10 about the ***AP*** – no ***AP*** for N09.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note. (Example 1)
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded).
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***). (Example 2)

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value. (Example 3)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts. (Example 3)

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp (**MR**) next to the total. Scoris will subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled **(d)** and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen). (Example 4)

10 Accuracy of Answers

*Candidates should **NOT** be penalized **IN THE N09 PAPER 1** for an accuracy error **(AP)**.*
For all questions, treat accuracy errors as though the ***AP*** has already been applied.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.
- Intermediate values are sometimes written as 3.24(741). This indicates that using 3.24 (or 3.25) is acceptable, but the more accurate value is 3.24741. The digits in brackets are not required for the marks. If candidates work with three or less significant figures, this could lead to an incorrect final value – in this case, do not award the final ***A1***.
- Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8).

If working is shown, and candidates work with more than three significant figures, once a correct value for the final answer is seen, do not penalize for subsequent rounding errors, or for not giving their answer correct to three significant figures.
e.g. if calculations lead to the value 2.8718, award the final ***A1*** for any of the following answers, which are to the incorrect level, and some are also incorrectly rounded, 2.871, 2.872, 2.8, 2.9, 2, 3. This is unlikely to happen on paper 1!

If there is no working shown, and answers are given to the correct two significant figures, award the ***N*** marks for correct two significant figures answers. However, do **not** accept answers to one significant figure without working

Below are the usual notes on applying the ***AP***. If you see work that falls into these categories, do **NOT** penalize for N09.

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct final answers not given to three significant figures.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 Style

The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Some of these examples may include ones using poor notation, to indicate what is acceptable. (Example 5)

EXAMPLES

Please check the references in the instructions above.

EXAMPLE 1

(a) evidence of using $\sum p_i = 1$ ***(M1)***

correct substitution ***A1***

e.g. $10k^2 + 3k + 0.6 = 1,\ 10k^2 + 3k - 0.4 = 0$

$k = 0.1$ ***A2*** ***N2***

Note: Award ***A1*** for a final answer of $k = 0.1,\ k = -0.4$.

(b) evidence of using $\mathrm{E}(X) = \sum p_i x_i$ ***(M1)***

correct substitution ***(A1)***

e.g. $-1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3$

$\mathrm{E}(X) = 1.5$ ***A1*** ***N2***

Note: Award ***FT*** marks only on values of *k* between 0 and 1.

EXAMPLE 2

(a) intercepts when $f(x) = 0$ ***(M1)***

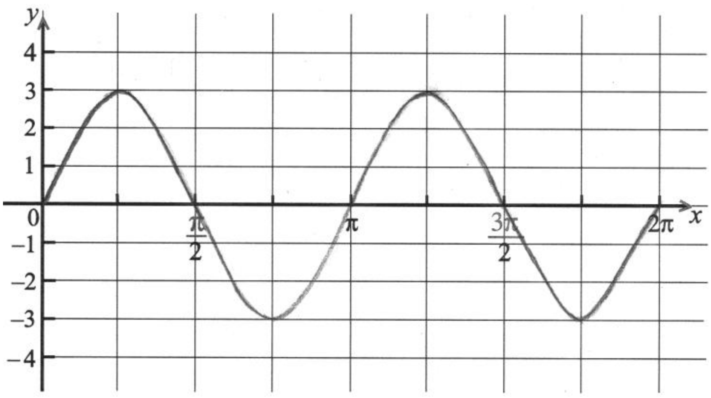
Note: 1 correct answer seen is sufficient evidence to award the ***(M1)***.

(1.54, 0) (4.13, 0) (accept $x = 1.54 \quad x = 4.13$) ***A1A1*** ***N3***

EXAMPLE 3

(a) period $= \pi$ (accept 180°) ***A1*** ***N1***

(b)



A1A1A1 ***N3***

Notes: Award ***A1*** for amplitude 3,
A1 for **their** period,
A1 for a sine curve through $(0,0)$ and $(0,2\pi)$.
If the answer to part (a) is incorrect, but the graph period is correct, award ***A1***.

(c) evidence of appropriate approach ***(M1)***
e.g. line $y = 2$ on graph, discussion of number of solutions in the domain
4 (solutions) ***A1*** ***N2***

Notes: Award the ***N2*** for 4 solutions, even if this is inconsistent with their graph.
Award ***M1A1 FT*** for incorrect answers which are consistent with their graph.

EXAMPLE 4

for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \ 2\cos(5x-3)\ \ 5 \ \ = 10\cos(5x-3)$ ***A1***

Award ***A1*** for $2\cos(5x-3)\ \ 5$, even if $10\cos(5x-3)$ is not seen.

EXAMPLE 5

(i) evidence of approach ***M1***
e.g. $\overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{AB}$, $B - A$

SECTION A

QUESTION 1

(a) (i) $g(0) = \mathrm{e}^0 - 2$ ***(A1)***

$= -1$ ***A1*** ***N2***

(ii) **METHOD 1**

substituting answer from (i) ***(M1)***

e.g. $(f \circ g)(0) = f(-1)$

correct substitution $f(-1) = 2(-1)^3 + 3$ ***(A1)***

$f(-1) = 1$ ***A1*** ***N3***

METHOD 2

attempt to find $(f \circ g)(x)$ ***(M1)***

e.g. $(f \circ g)(x) = f(\mathrm{e}^{3x} - 2) \quad = 2(\mathrm{e}^{3x} - 2)^3 + 3$

correct expression for $(f \circ g)(x)$ ***(A1)***

e.g. $2(\mathrm{e}^{3x} - 2)^3 + 3$

$(f \circ g)(0) = 1$ ***A1*** ***N3***

(b) interchanging x and y (seen anywhere) ***(M1)***

e.g. $x = 2y^3 + 3$

attempt to solve ***(M1)***

e.g. $y^3 = \dfrac{x-3}{2}$

$f^{-1}(x) = \sqrt[3]{\dfrac{x-3}{2}}$ ***A1*** ***N3***

[8 marks]

QUESTION 2

(a) evidence of equating scalar product to 0 ***(M1)***

$2 \times 3 + 3 \times (-1) + (-1) \times p = 0 \quad (6 - 3 - p = 0, 3 - p = 0)$ ***A1***

$p = 3$ ***A1*** ***N2***

(b) evidence of substituting into magnitude formula ***(M1)***

e.g. $\sqrt{1 + q^2 + 25}$, $1 + q^2 + 25$

setting up a correct equation ***A1***

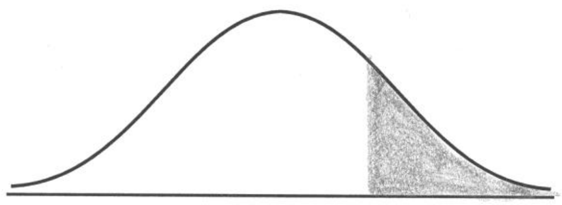
e.g. $\sqrt{1 + q^2 + 25} = \sqrt{42}$, $1 + q^2 + 25 = 42$, $q^2 = 16$

$q = \pm 4$ ***A1*** ***N2***

[6 marks]

QUESTION 3

(a)



A1A1 *N2*

Note: Award ***A1*** for vertical line to right of mean, ***A1*** for shading to right of **their** vertical line.

(b) evidence of recognizing symmetry *(M1)*

e.g. 105 is one standard deviation above the mean so d is one standard deviation below the mean, shading the corresponding part, $105-100=100-d$

$d=95$ *A1* *N2*

(c) evidence of using complement *(M1)*

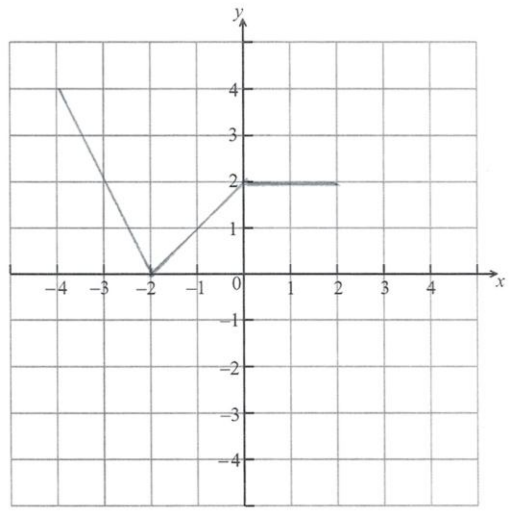
e.g. $1-0.32$, $1-p$

$\mathrm{P}(d<X<105)=0.68$ *A1* *N2*

[6 marks]

QUESTION 4

(a)



A2 *N2*

(b) evidence of appropriate approach *(M1)*

e.g. reference to any horizontal shift and/or stretch factor, $x=3+1$, $y=\frac{1}{2}\times 2$

P is $(4, 1)$ (accept $x=4$, $y=1$) *A1A1* *N3*

[5 marks]

QUESTION 5

(a) $f'(x) = 2x - \frac{p}{x^2}$ ***A1A1*** ***N2***

> **Note:** Award ***A1*** for $2x$, ***A1*** for $-\frac{p}{x^2}$.

(b) evidence of equating derivative to 0 (seen anywhere) ***(M1)***
evidence of finding $f'(-2)$ (seen anywhere) ***(M1)***
correct equation ***A1***
e.g. $-4 - \frac{p}{4} = 0$, $-16 - p = 0$

$p = -16$ ***A1*** ***N3***

[6 marks]

QUESTION 6

evidence of substituting for $\cos 2x$ ***(M1)***
evidence of substituting into $\sin^2 x + \cos^2 x = 1$ ***(M1)***
correct equation in terms of $\cos x$ (seen anywhere) ***A1***

e.g. $2\cos^2 x - 1 - 3\cos x - 3 = 1$, $2\cos^2 x - 3\cos x - 5 = 0$

evidence of appropriate approach to solve ***(M1)***
e.g. factorizing, quadratic formula

appropriate working ***A1***
e.g. $(2\cos x - 5)(\cos x + 1) = 0$, $(2x - 5)(x + 1)$, $\cos x = \frac{3 \pm \sqrt{49}}{4}$

correct solutions to the equation
e.g $\cos x = \frac{5}{2}$, $\cos x = -1$, $x = \frac{5}{2}$, $x = -1$ ***(A1)***
$x = \pi$ ***A1*** ***N4***

[7 marks]

QUESTION 7

(a) **METHOD 1**

recognizing that $f(8)=1$ ***(M1)***

e.g. $1=k\log_2 8$

recognizing that $\log_2 8=3$ ***(A1)***

e.g. $1=3k$

$k=\frac{1}{3}$ ***A1*** ***N2***

METHOD 2

attempt to find the inverse of $f(x)=k\log_2 x$ ***(M1)***

e.g. $x=k\log_2 y$, $y=2^{\frac{x}{k}}$

substituting 1 and 8 ***(M1)***

e.g. $1=k\log_2 8$, $2^{\frac{1}{k}}=8$

$k=\frac{1}{\log_2 8}$ $\left(k=\frac{1}{3}\right)$ ***A1*** ***N2***

(b) **METHOD 1**

recognizing that $f(x)=\frac{2}{3}$ ***(M1)***

e.g. $\frac{2}{3}=\frac{1}{3}\log_2 x$

$\log_2 x=2$ ***(A1)***

$f^{-1}\left(\frac{2}{3}\right)=4$ (accept $x=4$) ***A2*** ***N3***

METHOD 2

attempt to find inverse of $f(x)=\frac{1}{3}\log_2 x$ ***(M1)***

e.g. interchanging x and y, substituting $k=\frac{1}{3}$ into $y=2^{\frac{x}{k}}$

correct inverse ***(A1)***

e.g. $f^{-1}(x)=2^{3x}$, 2^{3x}

$f^{-1}\left(\frac{2}{3}\right)=4$ ***A2*** ***N3***

[7 marks]

SECTION B

QUESTION 8

(a) (i) evidence of substituting into $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ***(M1)***
e.g. $75 + 55 - 100$, Venn diagram

30 ***A1*** ***N2***

(ii) 45 ***A1*** ***N1***

[3 marks]

(b) (i) **METHOD 1**

evidence of using complement, Venn diagram ***(M1)***
e.g. $1 - p$, $100 - 30$

$\frac{70}{100} \quad \left(= \frac{7}{10}\right)$ ***A1*** ***N2***

METHOD 2

attempt to find $\mathrm{P}(\text{only one sport})$, Venn diagram ***(M1)***

e.g. $\frac{25}{100} + \frac{45}{100}$

$\frac{70}{100} \quad \left(= \frac{7}{10}\right)$ ***A1*** ***N2***

(ii) $\frac{45}{70} \quad \left(= \frac{9}{14}\right)$ ***A2*** ***N2***

[4 marks]

(c) valid reason in words or symbols ***(R1)***
e. g. $\mathrm{P}(A \cap B) = 0$ if mutually exclusive, $\mathrm{P}(A \cap B) \neq 0$ if not mutually exclusive

correct statement in words or symbols ***A1*** ***N2***
e.g. $\mathrm{P}(A \cap B) = 0.3$, $\mathrm{P}(A \cup B) \neq \mathrm{P}(A) + \mathrm{P}(B)$, $\mathrm{P}(A) + \mathrm{P}(B) > 1$, some students play both sports, sets intersect

[2 marks]

(d) valid reason for independence ***(R1)***
e.g. $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$, $\mathrm{P}(B \mid A) = \mathrm{P}(B)$

correct substitution ***A1A1*** ***N3***
e.g. $\frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}$, $\frac{30}{55} \neq \frac{75}{100}$

[3 marks]

Total [12 marks]

QUESTION 9

(a) (i) coordinates of A are $(0, -2)$ ***A1A1*** ***N2***

(ii) derivative of $x^2 - 4 = 2x$ (seen anywhere) ***(A1)***
evidence of correct approach ***(M1)***
e.g. quotient rule, chain rule

finding $f'(x)$ ***A2***

e.g. $f'(x) = 20 \times (-1) \times (x^2 - 4)^{-2} \times (2x)$, $\dfrac{(x^2 - 4)(0) - (20)(2x)}{(x^2 - 4)^2}$

substituting $x = 0$ into $f'(x)$ (do **not** accept solving $f'(x) = 0$) ***M1***
at A $f'(x) = 0$ ***AG*** ***N0***

[7 marks]

(b) (i) reference to $f'(x) = 0$ (seen anywhere) ***(R1)***
reference to $f''(0)$ is negative (seen anywhere) ***R1***
evidence of substituting $x = 0$ into $f''(x)$ ***M1***
finding $f''(0) = \dfrac{40 \times 4}{(-4)^3}$ $\left(= -\dfrac{5}{2}\right)$ ***A1***
then the graph must have a local maximum ***AG***

(ii) reference to $f''(x) = 0$ at point of inflexion, ***(R1)***
recognizing that the second derivative is never 0 ***A1*** ***N2***
e.g. $40(3x^2 + 4) \neq 0$, $3x^2 + 4 \neq 0$, $x^2 \neq -\dfrac{4}{3}$, the numerator is always positive

Note: Do **not** accept the use of the first derivative in part (b).

[6 marks]

(c) correct (informal) statement, including reference to approaching $y = 3$ ***A1*** ***N1***
e.g. getting closer to the line $y = 3$, horizontal asymptote at $y = 3$

[1 mark]

(d) **correct** inequalities, $y \leq -2$, $y > 3$, ***FT*** from (a)(i) and (c) ***A1A1*** ***N2***

[2 marks]

Total [16 marks]

QUESTION 10

(a) finding derivative ***(A1)***

e.g. $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$, $\frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) ***A1***

e.g. $\frac{1}{2\sqrt{4}}$, $\frac{1}{4}$

gradient of normal = $-\frac{1}{\text{gradient of tangent}}$ (seen anywhere) ***A1***

e.g. $-\frac{1}{f'(4)} = -4$, $-2\sqrt{x}$

substituting into equation of line (for normal) ***M1***

e.g. $y - 2 = -4(x - 4)$

$y = -4x + 18$ ***AG*** ***N0***

[4 marks]

(b) recognition that $y = 0$ at A ***(M1)***

e.g. $-4x + 18 = 0$

$x = \frac{18}{4}$ $\left(= \frac{9}{2}\right)$ ***A1*** ***N2***

[2 marks]

(c) splitting into two appropriate parts (areas and/or integrals) ***(M1)***

correct expression for area of R ***A2*** ***N3***

e.g. area of $R = \int_0^4 \sqrt{x}\,\mathrm{d}x + \int_4^{4.5} (-4x + 18)\,\mathrm{d}x$, $\int_0^4 \sqrt{x}\,\mathrm{d}x + \frac{1}{2} \times 0.5 \times 2$ (triangle)

Note: Award ***A1*** if dx is missing.

[3 marks]

continued…

Question 10 continued

(d) correct expression for the volume from $x = 0$ to $x = 4$ ***(A1)***

e.g. $V = \int_0^4 \pi\left[f(x)^2\right]\mathrm{d}x$, $\int_0^4 \pi\left(\sqrt{x}\right)^2 \mathrm{d}x$, $\int_0^4 \pi x \,\mathrm{d}x$

$V = \left[\frac{1}{2}\pi x^2\right]_0^4$ ***A1***

$V = \pi\left(\frac{1}{2}\times 16 - \frac{1}{2}\times 0\right)$ ***(A1)***

$V = 8\pi$ ***A1***

finding the volume from $x = 4$ to $x = 4.5$

EITHER

recognizing a cone ***(M1)***

e.g. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2 \times \frac{1}{2}$ ***(A1)***

$= \frac{2\pi}{3}$ ***A1***

total volume is $8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

OR

$V = \pi\int_4^{4.5}(-4x+18)^2\,\mathrm{d}x$ ***(M1)***

$= \int_4^{4.5}\pi(16x^2 - 144x + 324)\,\mathrm{d}x$

$= \pi\left[\frac{16}{3}x^3 - 72x^2 + 324x\right]_4^{4.5}$ ***A1***

$= \frac{2\pi}{3}$ ***A1***

total volume is $8\pi + \frac{2}{3}\pi \quad \left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

[8 marks]

Total [17 marks]