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International Baccalaureate®
Baccalauréat International
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MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 5 November 2009 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 8]*

Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.

(a) (i) Find $g(0)$.

(ii) Find $(f \circ g)(0)$. *[5 marks]*

(b) Find $f^{-1}(x)$. *[3 marks]*

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2. *[Maximum mark: 6]*

(a) Let $\boldsymbol{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{w}$, find the value of p. *[3 marks]*

(b) Let $\boldsymbol{v} = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix}$. Given that $|\boldsymbol{v}| = \sqrt{42}$, find the possible values of q. *[3 marks]*

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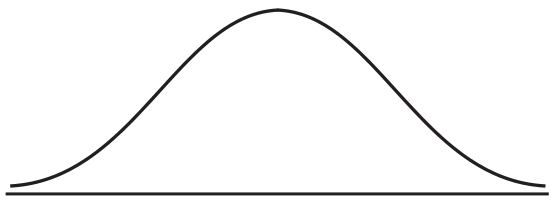
3. *[Maximum mark: 6]*

Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

(a) On the diagram below, shade the region representing $\mathrm{P}(X > 105)$. *[2 marks]*



(b) Given that $\mathrm{P}(X < d) = \mathrm{P}(X > 105)$, find the value of d. *[2 marks]*

(c) Given that $\mathrm{P}(X > 105) = 0.16$ (correct to two significant figures), find $\mathrm{P}(d < X < 105)$. *[2 marks]*

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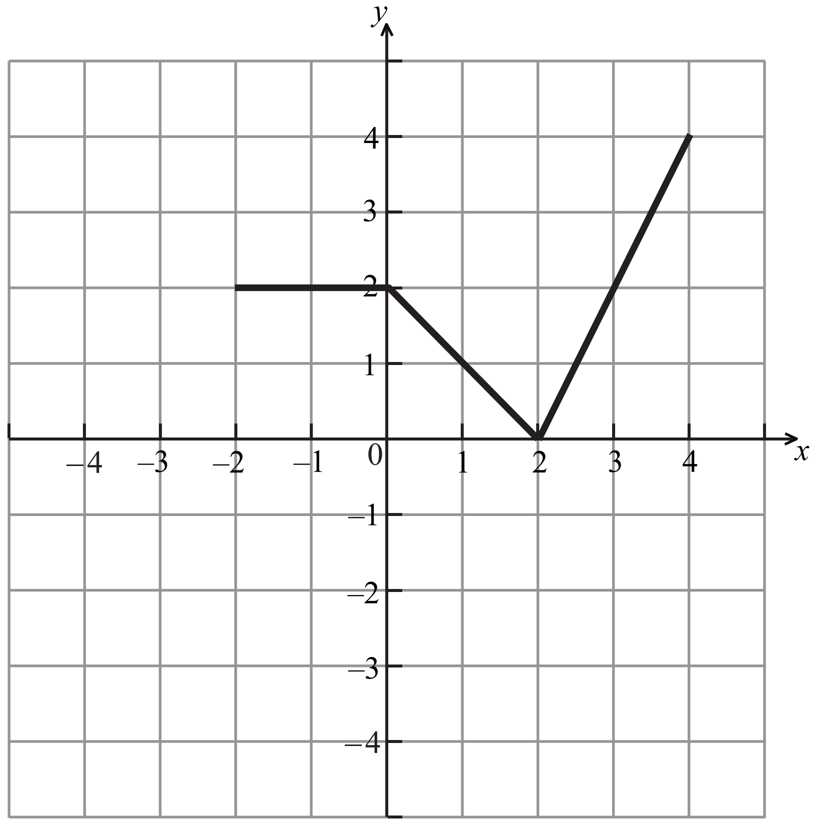
4. *[Maximum mark: 5]*

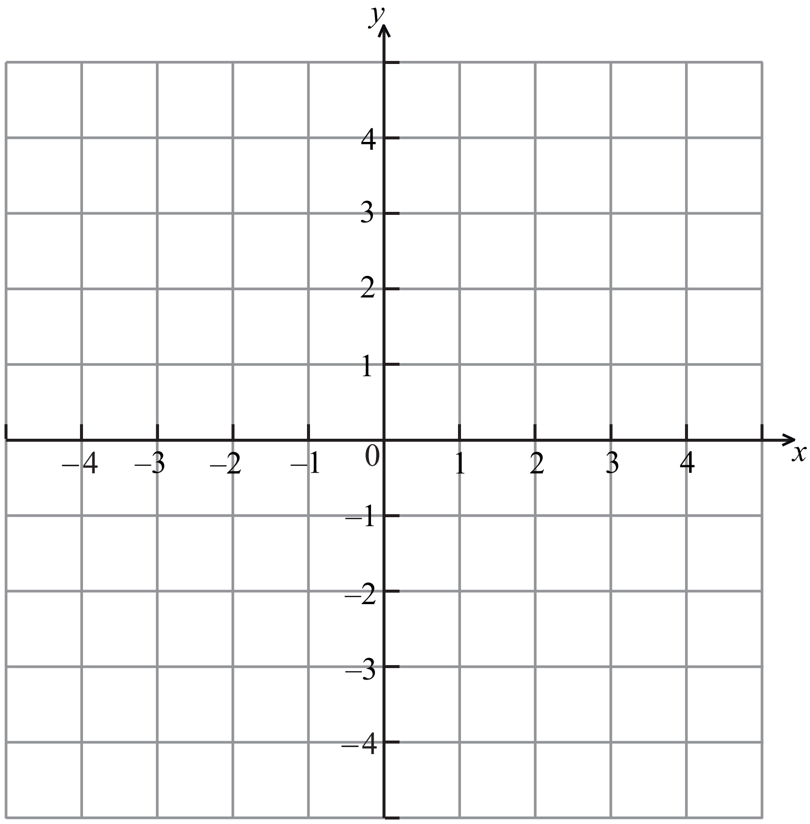
The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 4$.



(This question continues on the following page)

(Question 4 continued)

(a) Let $h(x) = f(-x)$. Sketch the graph of h on the grid below. *[2 marks]*



(b) Let $g(x) = \frac{1}{2} f(x-1)$. The point A$(3, 2)$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P. *[3 marks]*

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5. *[Maximum mark: 6]*

Consider $f(x) = x^2 + \frac{p}{x}$, $x \neq 0$, where p is a constant.

(a) Find $f'(x)$. *[2 marks]*

(b) There is a minimum value of $f(x)$ when $x = -2$. Find the value of p. *[4 marks]*

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6. *[Maximum mark: 7]*

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

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7. *[Maximum mark: 7]*

Let $f(x) = k \log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k. *[3 marks]*

(b) Find $f^{-1}\left(\frac{2}{3}\right)$. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 12]*

In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby. *[3 marks]*

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby. *[4 marks]*

Let A be the event that a boy plays football and B be the event that a boy plays rugby.

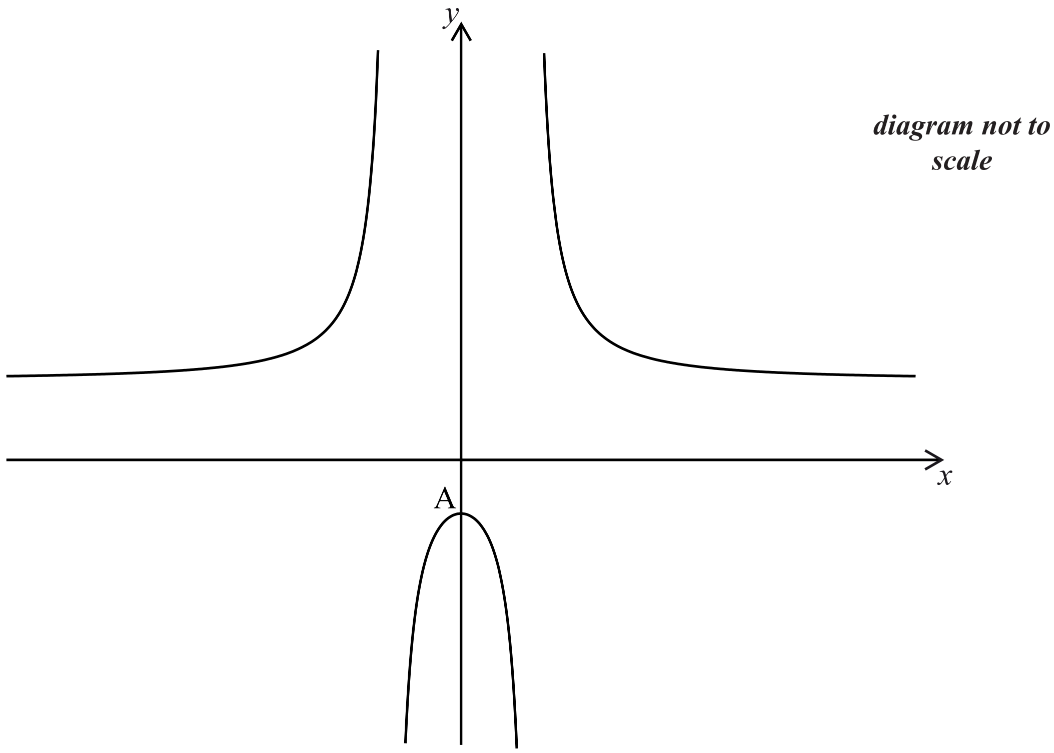
(c) Explain why A and B are **not** mutually exclusive. *[2 marks]*

(d) Show that A and B are **not** independent. *[3 marks]*

*Do **NOT** write on this page.*

9. *[Maximum mark: 16]*

Let $f(x) = 3 + \frac{20}{x^2 - 4}$, for $x \neq \pm 2$. The graph of f is given below.



The y-intercept is at the point A.

(a) (i) Find the coordinates of A.

(ii) Show that $f'(x) = 0$ at A. *[7 marks]*

(b) The second derivative $f''(x) = \frac{40(3x^2 + 4)}{(x^2 - 4)^3}$. Use this to

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does **not** have a point of inflexion. *[6 marks]*

(c) Describe the behaviour of the graph of f for large $|x|$. *[1 mark]*

(d) Write down the range of f. *[2 marks]*

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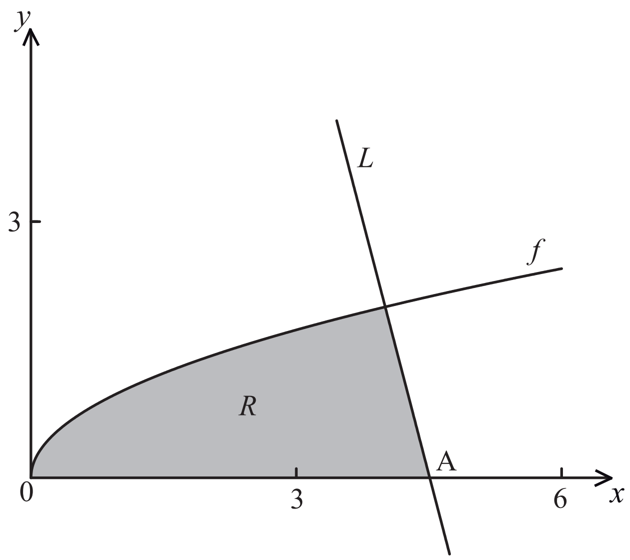
10. *[Maximum mark: 17]*

Let $f(x)=\sqrt{x}$. Line L is the normal to the graph of f at the point $(4, 2)$.

(a) Show that the equation of L is $y=-4x+18$. *[4 marks]*

(b) Point A is the x-intercept of L. Find the x-coordinate of A. *[2 marks]*

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L.



(c) Find an expression for the area of R. *[3 marks]*

(d) The region R is rotated 360° about the x-axis. Find the volume of the solid formed, giving your answer in terms of π. *[8 marks]*