



**MATHEMATICS
STANDARD LEVEL
PAPER 2**

Friday 8 May 2009 (morning)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

In an arithmetic series, the first term is -7 and the sum of the first 20 terms is 620.

(a) Find the common difference. [3 marks]

(b) Find the value of the 78th term. [2 marks]

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2. [Maximum mark: 7]

The circle shown has centre O and radius 3.9 cm.

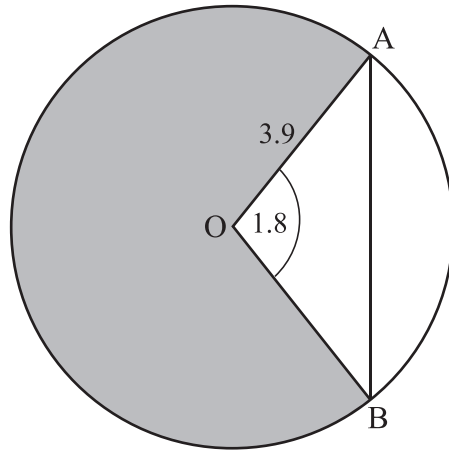


diagram not to scale

Points A and B lie on the circle and angle AOB is 1.8 radians.

(a) Find AB. [3 marks]

(b) Find the area of the shaded region. [4 marks]

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3. [Maximum mark: 6]

Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4 \cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

- (a) Find an expression for $h(x)$. [3 marks]
- (b) Write down the period of h . [1 mark]
- (c) Write down the range of h . [2 marks]

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4. [Maximum mark: 6]

A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $P(X \leq 475)$. [2 marks]

(b) Given that $P(X > a) = 0.27$, find a . [4 marks]

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5. [Maximum mark: 6]

Two lines with equations $\mathbf{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the

point P. Find the coordinates of P.

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6. [Maximum mark: 7]

In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

(a) Find the value of r . [3 marks]

(b) Find the smallest value of n for which $S_n > 40$. [4 marks]

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7. [Maximum mark: 8]

In any given season, a soccer team plays 65 % of their games at home.
When the team plays at home, they win 83 % of their games.
When they play away from home, they win 26 % of their games.

The team plays one game.

(a) Find the probability that the team wins the game. [4 marks]

(b) If the team does not win the game, find the probability that the game was played at home. [4 marks]

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SECTION B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

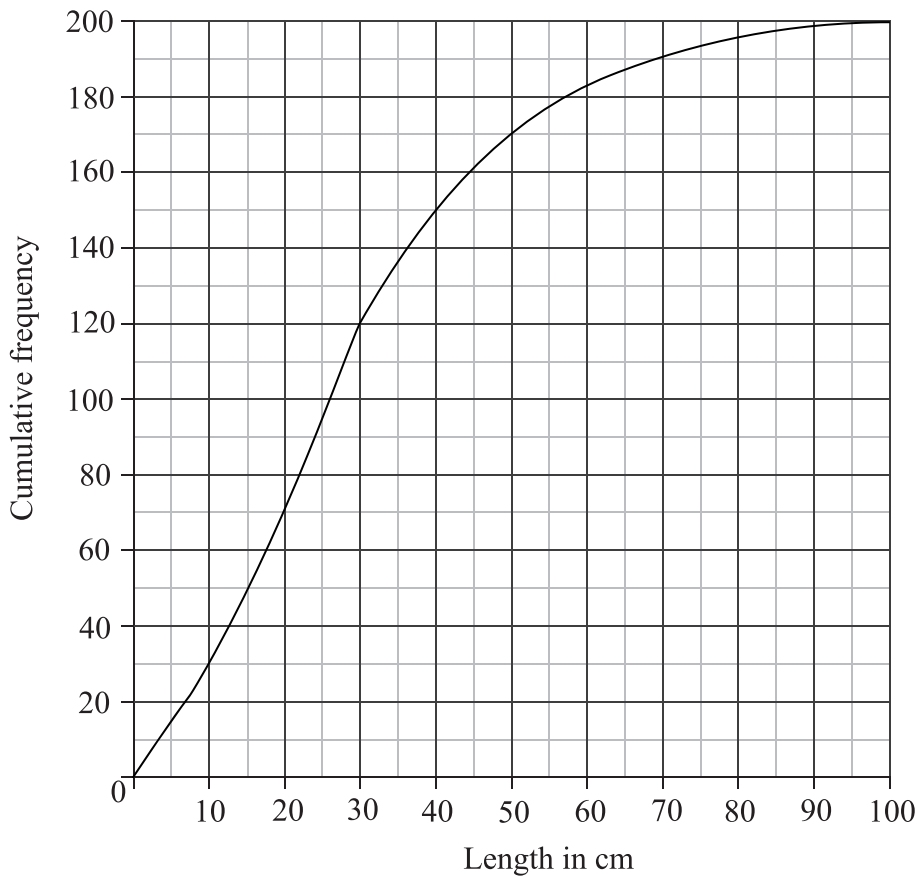
8. [Maximum mark: 15]

A fisherman catches 200 fish to sell. He measures the lengths, l cm of these fish, and the results are shown in the frequency table below.

Length l cm	$0 \leq l < 10$	$10 \leq l < 20$	$20 \leq l < 30$	$30 \leq l < 40$	$40 \leq l < 60$	$60 \leq l < 75$	$75 \leq l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish. [3 marks]

(b) A cumulative frequency diagram is given below for the lengths of the fish.



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(Question 8 (b) continued)

Use the graph to answer the following.

- (i) Estimate the interquartile range.
- (ii) Given that 40 % of the fish have a length more than k cm, find the value of k .

[6 marks]

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.
Medium fish have a length greater than or equal to 20 cm but less than 60 cm.
Large fish have a length greater than or equal to 60 cm.

- (c) Write down the probability that a fish is small.

[2 marks]

The cost of a small fish is \$4, a medium fish \$10, and a large fish \$12.

- (d) Copy and complete the following table, which gives a probability distribution for the cost \$ X .

[2 marks]

Cost \$$X$	4	10	12
$P(X = x)$		0.565	

- (e) Find $E(X)$.

[2 marks]



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9. [Maximum mark: 15]

Let $f(x) = ax^2 + bx + c$ where a , b and c are rational numbers.

- (a) The point $P(-4, 3)$ lies on the curve of f . Show that $16a - 4b + c = 3$. [2 marks]
- (b) The points $Q(6, 3)$ and $R(-2, -1)$ also lie on the curve of f . Write down two other linear equations in a , b and c . [2 marks]
- (c) These three equations may be written as a matrix equation in the form $AX = B$, where $X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.
 - (i) Write down the matrices A and B .
 - (ii) Write down A^{-1} .
 - (iii) **Hence** or otherwise, find $f(x)$. [8 marks]
- (d) Write $f(x)$ in the form $f(x) = a(x-h)^2 + k$, where a , h and k are rational numbers. [3 marks]

10. [Maximum mark: 15]

Let $f(x) = x^3 - 4x + 1$.

- (a) Expand $(x+h)^3$. [2 marks]
- (b) Use the formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$. [4 marks]
- (c) The tangent to the curve of f at the point $P(1, -2)$ is parallel to the tangent at a point Q . Find the coordinates of Q . [4 marks]
- (d) The graph of f is decreasing for $p < x < q$. Find the value of p and of q . [3 marks]
- (e) Write down the range of values for the gradient of f . [2 marks]

