22097305

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

**MATHEMATICS**
**STANDARD LEVEL**
**PAPER 1**

Thursday 7 May 2009 (afternoon)

1 hour 30 minutes

Candidate session number

| 0 | 0 | | | | | | | |
|---|---|---|---|---|---|---|---|---|

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

## SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

**1.** *[Maximum mark: 5]*

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

(a) Find $g^{-1}(x)$. *[2 marks]*

(b) Find $(f \circ g)(4)$. *[3 marks]*

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**2.** *[Maximum mark: 6]*

Find the cosine of the angle between the two vectors $3\boldsymbol{i}+4\boldsymbol{j}+5\boldsymbol{k}$ and $4\boldsymbol{i}-5\boldsymbol{j}-3\boldsymbol{k}$.

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**3.** *[Maximum mark: 6]*

The fifth term in the expansion of the binomial $(a+b)^n$ is given by $\binom{10}{4}p^6(2q)^4$.

(a) Write down the value of $n$. *[1 mark]*

(b) Write down $a$ and $b$, in terms of $p$ and/or $q$. *[2 marks]*

(c) Write down an expression for the sixth term in the expansion. *[3 marks]*

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**4.** *[Maximum mark: 5]*

(a) Find $\log_2 32$. *[1 mark]*

(b) Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of $p$ and of $q$. *[4 marks]*

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**5.** *[Maximum mark: 6]*

A matrix $\boldsymbol{M}$ has **inverse** $\boldsymbol{M}^{-1} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$.

(a) Find $\boldsymbol{M}$. *[3 marks]*

(b) Solve the matrix equation $\boldsymbol{MX} = \boldsymbol{B}$, where $\boldsymbol{B} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$. *[3 marks]*

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**6.** *[Maximum mark: 5]*

A function $f$ has its first derivative given by $f'(x) = (x-3)^3$.

(a) Find the second derivative. *[2 marks]*

(b) Find $f'(3)$ and $f''(3)$. *[1 mark]*

(c) The point P on the graph of $f$ has $x$-coordinate 3. Explain why P is not a point of inflexion. *[2 marks]*

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**7.** *[Maximum mark: 6]*

Let $f(x) = \sqrt{3}e^{2x}\sin x + e^{2x}\cos x$, for $0 \le x \le \pi$. Given that $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, solve the equation $f(x) = 0$.

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**8.** *[Maximum mark: 6]*

Let $f(x) = e^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

(a) Write down

(i) $f'(x)$;

(ii) $g'(x)$. *[2 marks]*

(b) Let $h(x) = e^{-3x}\sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. *[4 marks]*

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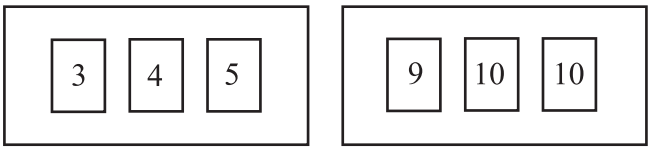
*Do **NOT** write on this page.*

## SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

**9.** *[Maximum mark: 12]*

Two boxes contain numbered cards as shown below.




Two cards are drawn at random, one from each box.

(a) Copy and complete the table below to show all nine equally likely outcomes. *[2 marks]*

| | | |
|---|---|---|
| 3 , 9 | | |
| 3 , 10 | | |
| 3 , 10 | | |

Let $S$ be the sum of the numbers on the two cards.

(b) Write down all the possible values of $S$. *[2 marks]*

(c) Find the probability of each value of $S$. *[2 marks]*

(d) Find the expected value of $S$. *[3 marks]*

(e) Anna plays a game where she wins \$50 if $S$ is even and loses \$30 if $S$ is odd. Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games. *[3 marks]*

*Do **NOT** write on this page.*

**10.** *[Maximum mark: 16]*

The line $L_1$ is parallel to the $z$-axis. The point P has position vector $\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}$ and lies on $L_1$.

(a) Write down the equation of $L_1$ in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

The line $L_2$ has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. The point A has position vector $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}$.

(b) Show that A lies on $L_2$. *[4 marks]*

Let B be the point of intersection of lines $L_1$ and $L_2$.

(c) (i) Show that $\overrightarrow{\text{OB}} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}$.

(ii) Find $\overrightarrow{\text{AB}}$. *[7 marks]*

(d) The point C is at $(2, 1, -4)$. Let D be the point such that ABCD is a parallelogram. Find $\overrightarrow{\text{OD}}$. *[3 marks]*

*Do **NOT** write on this page.*

**11.** *[Maximum mark: 17]*

*In this question s represents displacement in metres and t represents time in seconds.*

The velocity $v$ m s$^{-1}$ of a moving body is given by $v = 40 - at$ where $a$ is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for $s$ in terms of $a$ and $t$.

(ii) If $s = 0$ when $t = 0$, write down an expression for $s$ in terms of $a$ and $t$. *[6 marks]*

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of $a$.

(ii) Hence show that $a = \frac{8}{5}$. *[6 marks]*

(c) For a different train N, the value of $a$ is 4.
Show that this train will stop **before** it reaches the station. *[5 marks]*