



**MATHEMATICS
STANDARD LEVEL
PAPER 2**

Monday 10 November 2008 (morning)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Let $f(x) = 2x^2 + 4x - 6$.

- (a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$. [3 marks]
- (b) Write down the equation of the axis of symmetry of the graph of f . [1 mark]
- (c) Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$. [2 marks]

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2. [Maximum mark: 6]

(a) Expand $(x-2)^4$ and simplify your result. [3 marks]

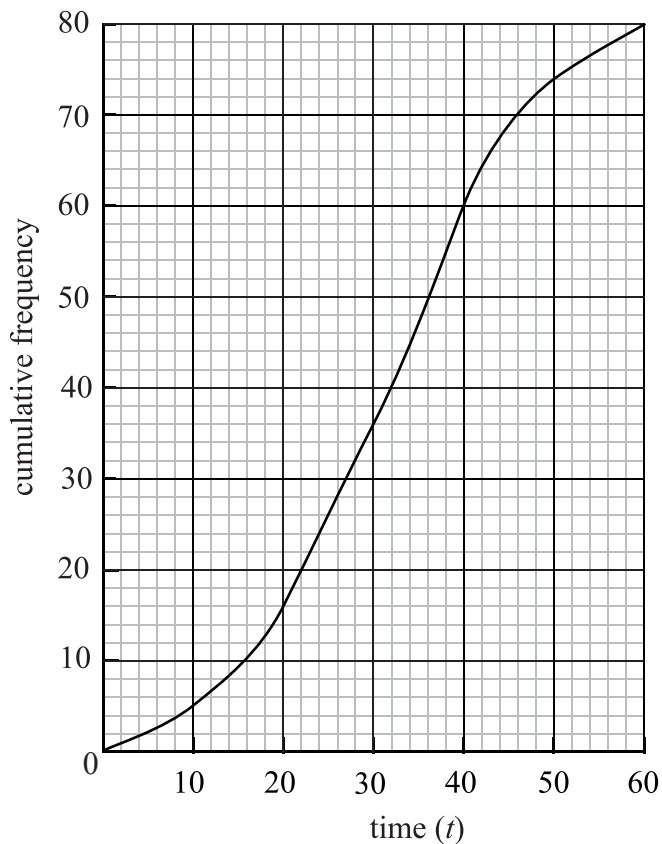
(b) Find the term in x^3 in $(3x+4)(x-2)^4$. [3 marks]

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3. [Maximum mark: 6]

The following is a cumulative frequency diagram for the time t , in minutes, taken by 80 students to complete a task.



(a) Write down the median. [1 mark]

(b) Find the interquartile range. [3 marks]

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(This question continues on the following page)



(Question 3 continued)

(c) Complete the frequency table below.

[2 marks]

Time (minutes)	Number of students
$0 \leq t < 10$	5
$10 \leq t < 20$	
$20 \leq t < 30$	20
$30 \leq t < 40$	24
$40 \leq t < 50$	
$50 \leq t < 60$	6

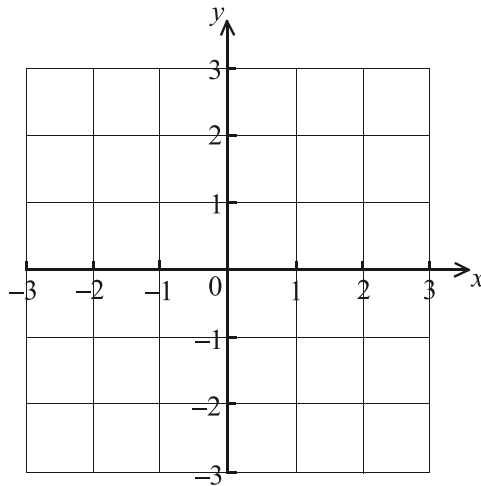


4. [Maximum mark: 8]

Let $f(x) = x \cos(x - \sin x)$, $0 \leq x \leq 3$.

(a) Sketch the graph of f on the following set of axes.

[3 marks]



(b) The graph of f intersects the x -axis when $x = a$, $a \neq 0$. Write down the value of a .

[1 mark]

(c) The graph of f is revolved 360° about the x -axis from $x = 0$ to $x = a$. Find the volume of the solid formed.

[4 marks]

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5. [Maximum mark: 5]

The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads. [2 marks]

(b) Find the probability of obtaining **at least** two heads. [3 marks]

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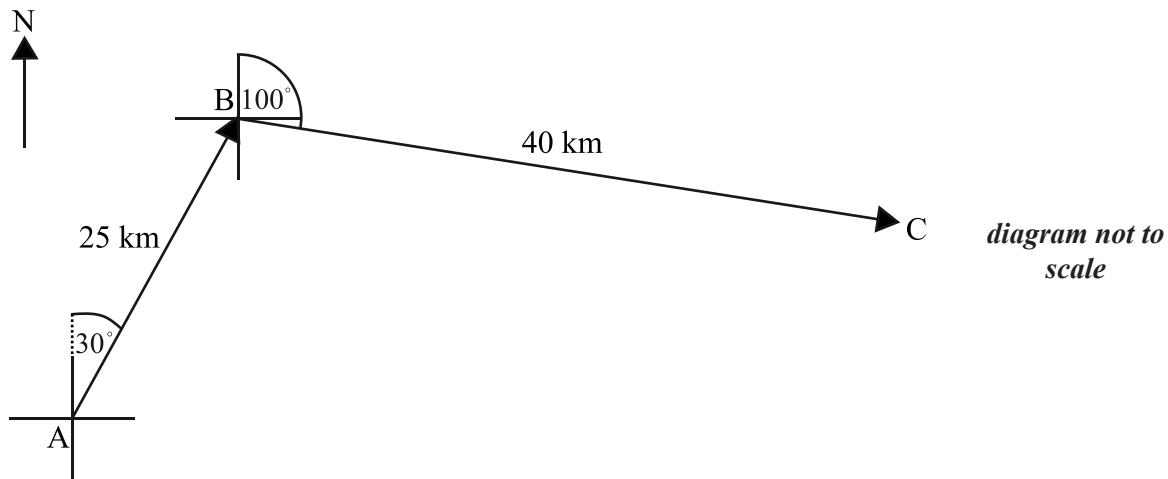
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6. [Maximum mark: 7]

A ship leaves port A on a bearing of 030° . It sails a distance of 25 km to point B. At B, the ship changes direction to a bearing of 100° . It sails a distance of 40 km to reach point C. This information is shown in the diagram below.



A second ship leaves port A and sails directly to C.

- (a) Find the distance the second ship will travel. [4 marks]
- (b) Find the bearing of the course taken by the second ship. [3 marks]

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7. [Maximum mark: 7]

The scores of a test given to students are normally distributed with a mean of 21. 80 % of the students have scores less than 23.7.

- (a) Find the standard deviation of the scores. [3 marks]

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than b .

- (b) (i) Find the probability the student has a score less than 25.4.
- (ii) Find the value of b . [4 marks]

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SECTION B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 18]

The diagram shows a parallelogram ABCD.

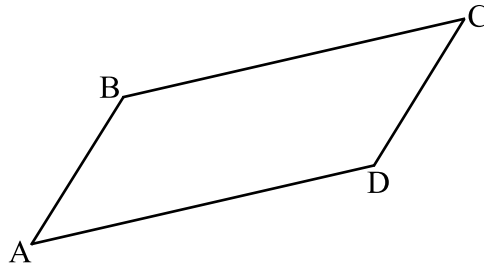


diagram not to scale

The coordinates of A, B and D are A(1, 2, 3), B(6, 4, 4) and D(2, 5, 5).

- (a) (i) Show that $\vec{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$.
- (ii) Find \vec{AD} .
- (iii) **Hence** show that $\vec{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$. [5 marks]
- (b) Find the coordinates of point C. [3 marks]
- (c) (i) Find $\vec{AB} \cdot \vec{AD}$.
- (ii) **Hence** find angle A. [7 marks]
- (d) Hence, or otherwise, find the area of the parallelogram. [3 marks]



9. [Maximum mark: 14]

Let $f(x) = e^{2x} \cos x$, $-1 \leq x \leq 2$.

(a) Show that $f'(x) = e^{2x}(2 \cos x - \sin x)$. [3 marks]

Let the line L be the normal to the curve of f at $x = 0$.

(b) Find the equation of L . [5 marks]

The graph of f and the line L intersect at the point $(0, 1)$ and at a second point P .

(c) (i) Find the x -coordinate of P .
(ii) Find the area of the region **enclosed** by the graph of f and the line L . [6 marks]



10. [Maximum mark: 13]

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}.$$

(a) Write down \mathbf{A}^{-1} . [2 marks]

The matrix \mathbf{B} satisfies the equation $\left(\mathbf{I} - \frac{1}{2}\mathbf{B}\right)^{-1} = \mathbf{A}$, where \mathbf{I} is the 3×3 identity matrix.

- (b) (i) Show that $\mathbf{B} = -2(\mathbf{A}^{-1} - \mathbf{I})$.
- (ii) Find \mathbf{B} .
- (iii) Write down $\det \mathbf{B}$.
- (iv) **Hence**, explain why \mathbf{B}^{-1} exists. [6 marks]

$$\text{Let } \mathbf{B}\mathbf{X} = \mathbf{C}, \text{ where } \mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.$$

- (c) (i) Find \mathbf{X} .
- (ii) Write down a system of equations whose solution is represented by \mathbf{X} . [5 marks]

