IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

N07/5/MATME/SP2/ENG/TZ0/XX+

88077302

MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 6 November 2007 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Total mark: 24]*

Part A *[Maximum mark: 13]*

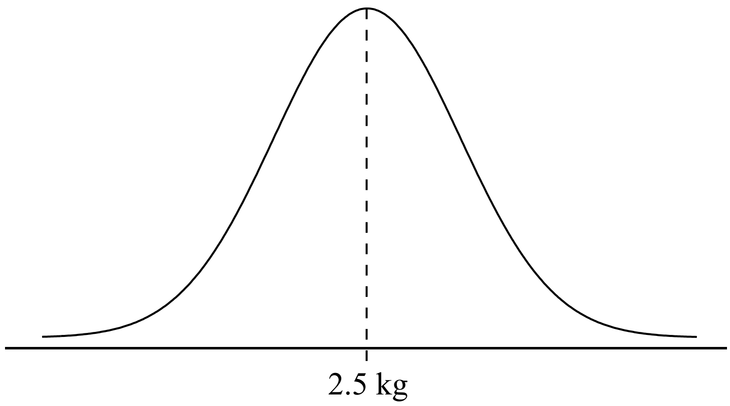
The weights of chickens for sale in a shop are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

(a) A chicken is chosen at random.

(i) Find the probability that it weighs less than 2 kg.

(ii) Find the probability that it weighs more than 2.8 kg.

(iii) Copy the diagram below. Shade the areas that represent the probabilities from parts (i) and (ii).



(iv) **Hence** show that the probability that it weighs between 2 kg and 2.8 kg is 0.7936 (to four significant figures). *[7 marks]*

(b) A customer buys 10 chickens.

(i) Find the probability that all 10 chickens weigh between 2 kg and 2.8 kg.

(ii) Find the probability that at least 7 of the chickens weigh between 2 kg and 2.8 kg. *[6 marks]*

(This question continues on the following page)

(Question 1 continued)

Part B *[Maximum mark: 11]*

Let $\boldsymbol{A} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

(a) Find

(i) $\boldsymbol{A}^{-1}$;

(ii) $\boldsymbol{A}^{2}$. *[4 marks]*

Let $\boldsymbol{B} = \begin{pmatrix} p & 2 \\ 0 & q \end{pmatrix}$.

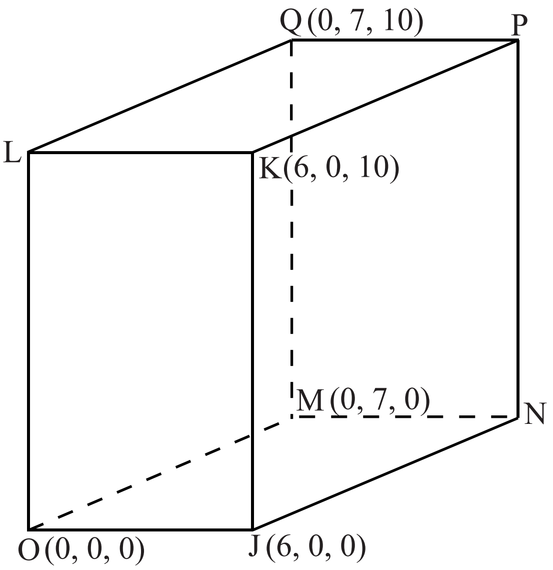
(b) Given that $2\boldsymbol{A} + \boldsymbol{B} = \begin{pmatrix} 2 & 6 \\ 4 & 3 \end{pmatrix}$, find the value of p and of q. *[3 marks]*

(c) Hence find $\boldsymbol{A}^{-1}\boldsymbol{B}$. *[2 marks]*

(d) Let $\boldsymbol{X}$ be a 2×2 matrix such that $\boldsymbol{AX} = \boldsymbol{B}$. Find $\boldsymbol{X}$. *[2 marks]*

2. *[Maximum mark: 16]*

The diagram below shows a cuboid (rectangular solid) OJKLMNPQ. The vertex O is $(0, 0, 0)$, J is $(6, 0, 0)$, K is $(6, 0, 10)$, M is $(0, 7, 0)$ and Q is $(0, 7, 10)$.



(a) (i) Show that $\overrightarrow{JQ} = \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$.

(ii) Find $\overrightarrow{MK}$. *[2 marks]*

(b) An equation for the line (MK) is $\boldsymbol{r} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s\begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$.

(i) Write down an equation for the line (JQ) in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Find the acute angle between (JQ) and (MK). *[9 marks]*

(c) The lines (JQ) and (MK) intersect at D. Find the position vector of D. *[5 marks]*

3. *[Maximum mark: 17]*

Consider the function $f(x) = e^{(2x-1)} + \frac{5}{(2x-1)}$, $x \neq \frac{1}{2}$.

(a) Sketch the curve of f for $-2 \leq x \leq 2$, including any asymptotes. *[3 marks]*

(b) (i) Write down the equation of the vertical asymptote of f.

(ii) Write down which one of the following expressions does **not** represent an area between the curve of f and the x-axis.

$$\int_1^2 f(x)\,dx$$

$$\int_0^2 f(x)\,dx$$

(iii) Justify your answer. *[3 marks]*

(c) The region between the curve and the x-axis between $x = 1$ and $x = 1.5$ is rotated through 360° about the x-axis. Let V be the volume formed.

(i) Write down an expression to represent V.

(ii) Hence write down the value of V. *[4 marks]*

(d) Find $f'(x)$. *[4 marks]*

(e) (i) Write down the value of x at the minimum point on the curve of f.

(ii) The equation $f(x) = k$ has no solutions for $p \leq k < q$. Write down the value of p and of q. *[3 marks]*

4. *[Maximum mark: 11]*

(a) Consider the equation $4x^2 + kx + 1 = 0$. For what values of k does this equation have two **equal** roots? *[3 marks]*

Let f be the function $f(\theta) = 2\cos 2\theta + 4\cos\theta + 3$, for $-360^\circ \le \theta \le 360^\circ$.

(b) Show that this function may be written as $f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$. *[1 mark]*

(c) Consider the equation $f(\theta) = 0$, for $-360^\circ \le \theta \le 360^\circ$.

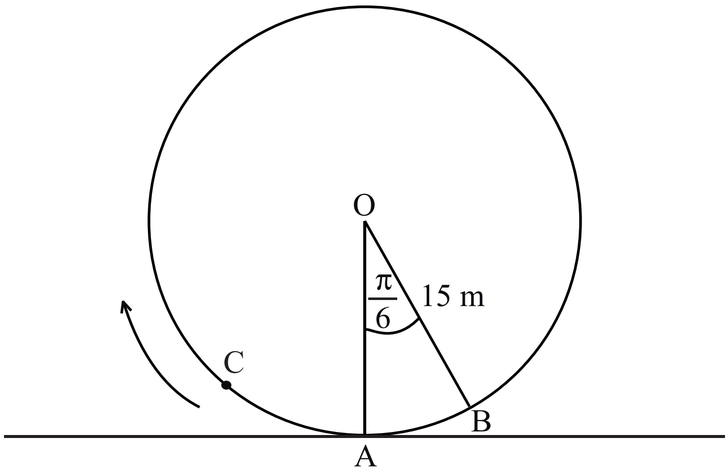
(i) How many distinct values of $\cos\theta$ satisfy this equation?

(ii) Find all values of θ which satisfy this equation. *[5 marks]*

(d) Given that $f(\theta) = c$ is satisfied by only three values of θ, find the value of c. *[2 marks]*

5. *[Maximum mark: 22]*

A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram below. Initially seat A is at ground level. The next seat is B, where $\hat{AOB} = \frac{\pi}{6}$.



(a) Find the length of the arc AB. *[2 marks]*

(b) Find the area of the sector AOB. *[2 marks]*

(c) The wheel turns clockwise through an angle of $\frac{2\pi}{3}$. Find the height of A above the ground. *[3 marks]*

The height, h metres, of seat C above the ground after t minutes, can be modelled by the function

$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right).$$

(d) (i) Find the height of seat C when $t = \frac{\pi}{4}$.

(ii) Find the initial height of seat C.

(iii) Find the time at which seat C first reaches its highest point. *[8 marks]*

(e) Find $h'(t)$. *[2 marks]*

(f) For $0 \le t \le \pi$,

(i) sketch the graph of h';

(ii) find the time at which the height is changing most rapidly. *[5 marks]*