

88077301

MATHEMATICS
STANDARD LEVEL
PAPER 1

Monday 5 November 2007 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

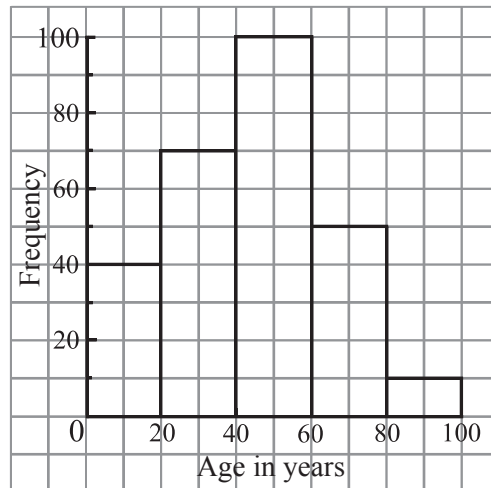
- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The histogram below represents the ages of 270 people in a village.



(a) Use the histogram to complete the table below.

[2 marks]

Age range	Frequency	Mid-interval value
$0 \leq \text{age} < 20$	40	10
$20 \leq \text{age} < 40$		
$40 \leq \text{age} < 60$		
$60 \leq \text{age} < 80$		
$80 \leq \text{age} \leq 100$		

(b) Hence, calculate an estimate of the mean age.

[4 marks]

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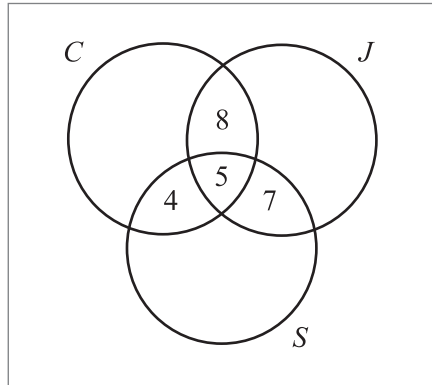
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2. [Maximum mark: 6]

The Venn diagram below shows information about 120 students in a school. Of these, 40 study Chinese (C), 35 study Japanese (J), and 30 study Spanish (S).



A student is chosen at random from the group. Find the probability that the student

(a) studies exactly two of these languages; [1 mark]

(b) studies only Japanese; [2 marks]

(c) does not study any of these languages. [3 marks]

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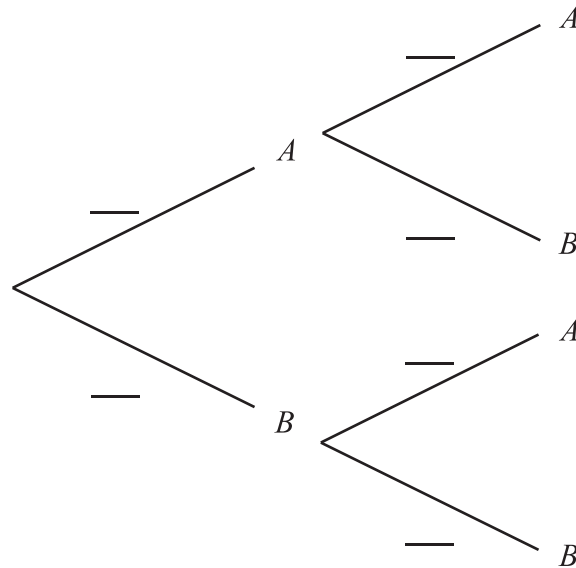
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3. [Maximum mark: 6]

A bag contains four apples (A) and six bananas (B). A fruit is taken from the bag and eaten. Then a second fruit is taken and eaten.

- (a) Complete the tree diagram below by writing probabilities in the spaces provided. [3 marks]



- (b) Find the probability that one of each type of fruit was eaten. [3 marks]

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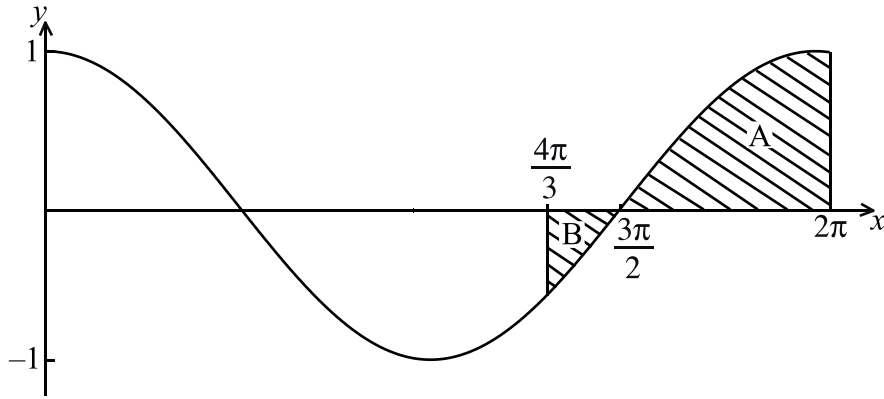
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4. [Maximum mark: 6]

The following diagram shows part of the graph of $y = \cos x$ for $0 \leq x \leq 2\pi$. Regions A and B are shaded.



- (a) Write down an expression for the area of A. [1 mark]
- (b) Calculate the area of A. [1 mark]
- (c) Find the total area of the shaded regions. [4 marks]

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5. [Maximum mark: 6]

The first four terms of a sequence are 18, 54, 162, 486.

(a) Use all four terms to show that this is a geometric sequence. [2 marks]

(b) (i) Find an expression for the n^{th} term of this geometric sequence.

(ii) If the n^{th} term of the sequence is 1 062 882, find the value of n . [4 marks]

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6. [Maximum mark: 6]

(a) Write down the first three terms of the sequence $u_n = 3n$, for $n \geq 1$. [1 mark]

(b) Find

(i) $\sum_{n=1}^{20} 3n$;

(ii) $\sum_{n=21}^{100} 3n$. [5 marks]

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7. [Maximum mark: 6]

Let $f(x) = \log_a x$, $x > 0$.

(a) Write down the value of

(i) $f(a)$;

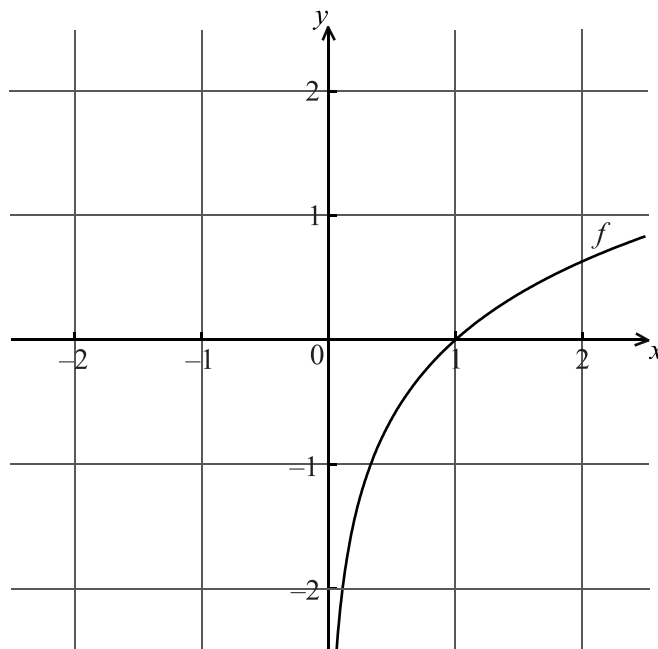
(ii) $f(1)$;

(iii) $f(a^4)$.

[3 marks]

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(b) The diagram below shows part of the graph of f .



On the same diagram, sketch the graph of f^{-1} .

[3 marks]



8. [Maximum mark: 6]

Consider the function $f(x) = 4x^3 + 2x$. Find the equation of the normal to the curve of f at the point where $x = 1$.

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9. [Maximum mark: 6]

Differentiate each of the following with respect to x .

(a) $y = \sin 3x$ [1 mark]

(b) $y = x \tan x$ [2 marks]

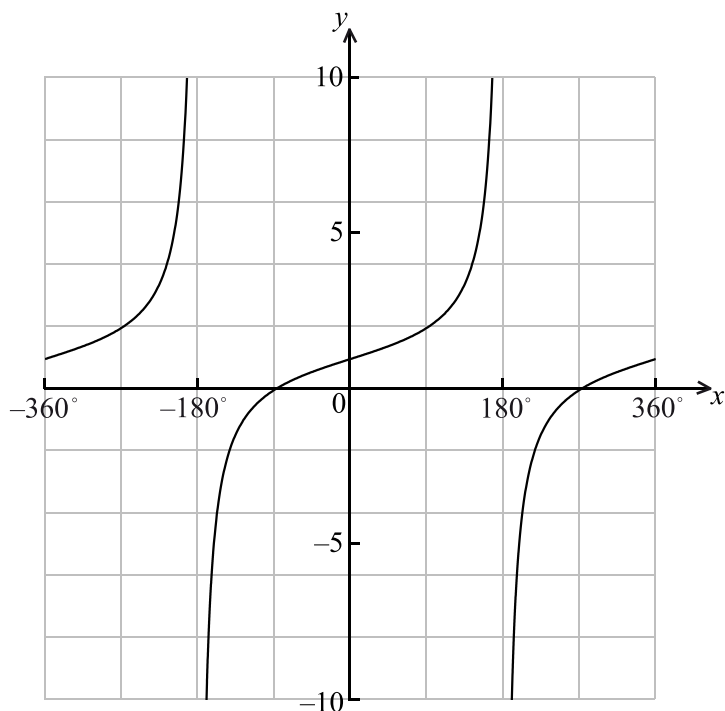
(c) $y = \frac{\ln x}{x}$ [3 marks]

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10. [Maximum mark: 6]

The diagram below shows the graph of $f(x) = 1 + \tan\left(\frac{x}{2}\right)$ for $-360^\circ \leq x \leq 360^\circ$.



- (a) On the same diagram, draw the asymptotes. [2 marks]
- (b) Write down
 - (i) the period of the function;
 - (ii) the value of $f(90^\circ)$. [2 marks]
- (c) Solve $f(x) = 0$ for $-360^\circ \leq x \leq 360^\circ$. [2 marks]

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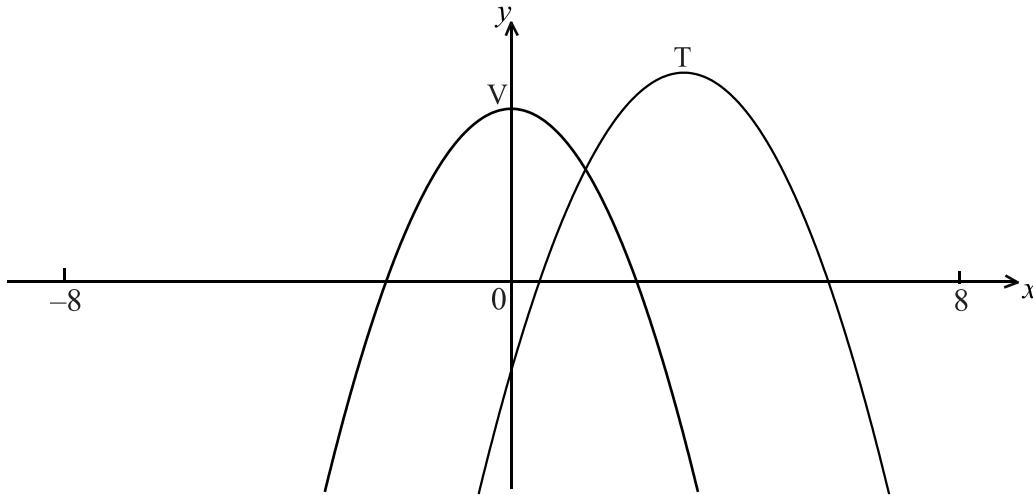
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11. [Maximum mark: 6]

The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex $V(0, 5)$.

Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex $T(3, 6)$.



(a) Write down the value of

(i) h ;

(ii) k .

[2 marks]

(b) Write down an expression for $g(x)$.

[2 marks]

(c) On the same diagram, sketch the graph of $y = g(-x)$.

[2 marks]

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12. [Maximum mark: 6]

A discrete random variable X has a probability distribution as shown in the table below.

x	0	1	2	3
$P(X = x)$	0.1	a	0.3	b

(a) Find the value of $a + b$. [2 marks]

(b) Given that $E(X) = 1.5$, find the value of a and of b . [4 marks]

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13. [Maximum mark: 6]

(a) Expand $\left(e + \frac{1}{e}\right)^4$ in terms of e . [4 marks]

(b) Express $\left(e + \frac{1}{e}\right)^4 + \left(e - \frac{1}{e}\right)^4$ as the sum of three terms. [2 marks]

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14. [Maximum mark: 6]

The area $A \text{ km}^2$ affected by a forest fire at time t hours is given by $A = A_0 e^{kt}$.
When $t = 5$, the area affected is 1 km^2 and the rate of change of the area is $0.2 \text{ km}^2 \text{ h}^{-1}$.

- (a) Show that $k = 0.2$. [4 marks]

- (b) Given that $A_0 = \frac{1}{e}$, find the value of t when 100 km^2 are affected. [2 marks]

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15. [Maximum mark: 6]

On the axes below, sketch a curve $y = f(x)$ which satisfies the following conditions.

x	$f(x)$	$f'(x)$	$f''(x)$
$-2 \leq x < 0$		negative	positive
0	-1	0	positive
$0 < x < 1$		positive	positive
1	2	positive	0
$1 < x \leq 2$		positive	negative

