IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
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M07/5/MATME/SP2/ENG/TZ2/XX

22077306

MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 8 May 2007 (morning)

1 hour 30 minutes

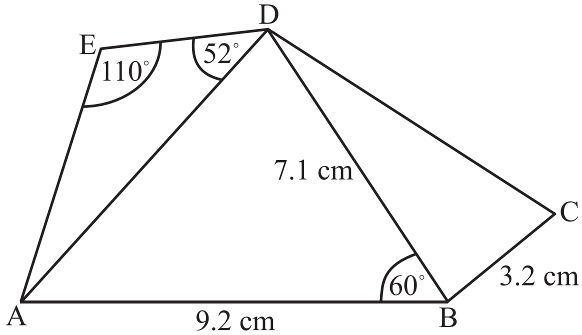
INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 21]*

The following diagram shows a pentagon ABCDE, with $\text{AB} = 9.2\text{ cm}$, $\text{BC} = 3.2\text{ cm}$, $\text{BD} = 7.1\text{ cm}$, $\text{A}\hat{\text{E}}\text{D} = 110^\circ$, $\text{A}\hat{\text{D}}\text{E} = 52^\circ$ and $\text{A}\hat{\text{B}}\text{D} = 60^\circ$.



(a) Find AD. *[4 marks]*

(b) Find DE. *[4 marks]*

(c) The area of triangle BCD is 5.68 cm^2. Find $\text{D}\hat{\text{B}}\text{C}$. *[4 marks]*

(d) Find AC. *[4 marks]*

(e) Find the area of quadrilateral ABCD. *[5 marks]*

2. *[Maximum mark: 12]*

There are 50 boxes in a factory. Their weights, w kg, are divided into 5 classes, as shown in the following table.

Class	Weight (kg)	Number of boxes
A	$9.5 \le w < 18.5$	7
B	$18.5 \le w < 27.5$	12
C	$27.5 \le w < 36.5$	13
D	$36.5 \le w < 45.5$	10
E	$45.5 \le w < 54.5$	8

(a) Show that the estimated mean weight of the boxes is 32 kg. *[3 marks]*

(b) There are x boxes in the factory marked "Fragile". They are all in class E. The estimated mean weight of all the other boxes in the factory is 30 kg. Calculate the value of x. *[4 marks]*

(c) An additional y boxes, all with a weight in class D, are delivered to the factory. The total estimated mean weight of **all** of the boxes in the factory is less than 33 kg. Find the largest possible value of y. *[5 marks]*

3. *[Maximum mark: 17]*

In this question, distance is in metres, time is in minutes.

Two model airplanes are each flying in a straight line.

At 13:00 the first model airplane is at the point $(3, 2, 7)$. Its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$.

(a) Find the speed of the model airplane. *[2 marks]*

At 13:00 the second model airplane is at the point $(-5, 10, 23)$. After two minutes, it is at the point $(3, 16, 39)$.

(b) Show that its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$. *[3 marks]*

(c) The airplanes meet at point Q.

(i) At what time do the airplanes meet?

(ii) Find the position of Q. *[6 marks]*

(d) Find the angle θ between the paths of the two airplanes. *[6 marks]*

4. *[Maximum mark: 16]*

Two restaurants, *Center* and *New*, sell fish rolls and salads.

Let F be the event a customer chooses a fish roll.
Let S be the event a customer chooses a salad.
Let N be the event a customer chooses neither a fish roll nor a salad.

In the *Center* restaurant $\mathrm{P}(F) = 0.31$, $\mathrm{P}(S) = 0.62$, $\mathrm{P}(N) = 0.14$.

(a) Show that $\mathrm{P}(F \cap S) = 0.07$. *[3 marks]*

(b) Given that a customer chooses a salad, find the probability the customer also chooses a fish roll. *[3 marks]*

(c) Are F and S independent events? Justify your answer. *[3 marks]*

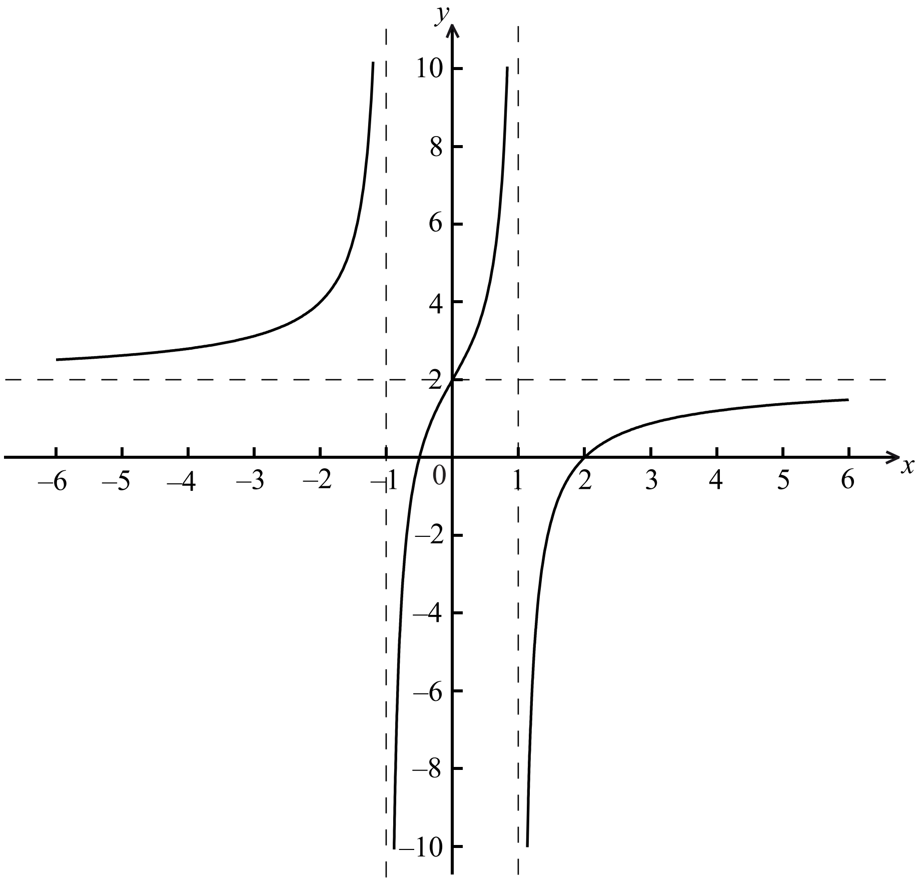
At *New* restaurant, $\mathrm{P}(N) = 0.14$. Twice as many customers choose a salad as choose a fish roll. Choosing a fish roll is **independent** of choosing a salad.

(d) Find the probability that a fish roll is chosen. *[7 marks]*

5. *[Maximum mark: 24]*

Let $f(x) = p - \dfrac{3x}{x^2 - q^2}$, where $p,\ q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptotes, is shown below.



(a) The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$. Write down the value of

(i) p;

(ii) q. *[2 marks]*

(b) Let R be the region bounded by the graph of f, the x-axis, and the y-axis.

(i) Find the negative x-intercept of f.

(ii) Hence find the volume obtained when R is revolved through 360° about the x-axis. *[7 marks]*

(This question continues on the following page)

(Question 5 continued)

(c) (i) Show that $f'(x) = \dfrac{3(x^2+1)}{(x^2-1)^2}$.

(ii) Hence, show that there are no maximum or minimum points on the graph of f. *[8 marks]*

(d) Let $g(x) = f'(x)$. Let A be the area of the region enclosed by the graph of g and the x-axis, between $x = 0$ and $x = a$, where $a > 0$. Given that $A = 2$, find the value of a. *[7 marks]*
