IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

N06/5/MATME/SP2/ENG/TZ0/XX

88067302

MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 3 November 2006 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

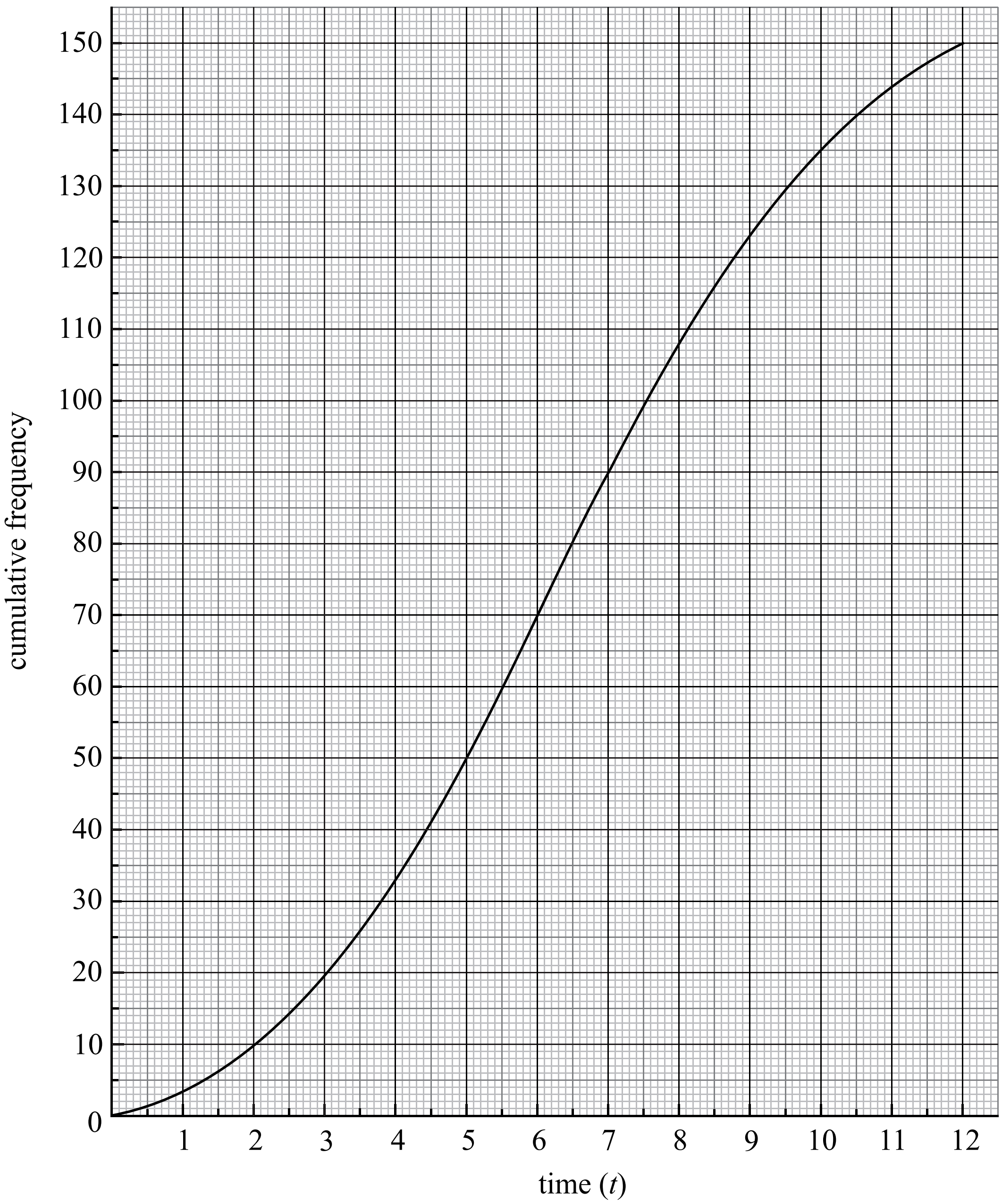
- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Total mark: 24]*

Part A *[Maximum mark: 14]*

The following is the cumulative frequency curve for the time, t minutes, spent by 150 people in a store on a particular day.



(This question continues on the following page)

(Question 1 continued)

(a) (i) How many people spent less than 5 minutes in the store?

(ii) Find the number of people who spent between 5 and 7 minutes in the store.

(iii) Find the median time spent in the store. *[6 marks]*

(b) Given that 40 % of the people spent longer than k minutes, find the value of k. *[3 marks]*

(c) (i) **On your answer sheet**, copy and complete the following frequency table.

t (minutes)	$0 \le t < 2$	$2 \le t < 4$	$4 \le t < 6$	$6 \le t < 8$	$8 \le t < 10$	$10 \le t < 12$
Frequency	10	23				15

(ii) Hence, calculate an estimate for the mean time spent in the store. *[5 marks]*

Part B *[Maximum mark: 10]*

Two fair **four**-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

(a) Write down

(i) the sample space;

(ii) the probability that two scores of 4 are obtained. *[4 marks]*

Let X be the number of 4s that land face down.

(b) **Copy** and complete the following probability distribution for X.

x	0	1	2
$\mathrm{P}(X = x)$			

[3 marks]

(c) Find $\mathrm{E}(X)$. *[3 marks]*

2. *[Maximum mark: 14]*

The function f is given by $f(x) = mx^3 + nx^2 + px + q$, where m, n, p, q are integers. The graph of f passes through the point (0, 0).

(a) Write down the value of q. *[1 mark]*

The graph of f also passes through the point (3, 18).

(b) Show that $27m + 9n + 3p = 18$. *[2 marks]*

The graph of f also passes through the points (1, 0) and (–1, –10).

(c) Write down the other two linear equations in m, n and p. *[2 marks]*

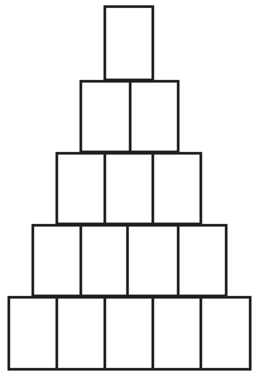
(d) (i) Write down these three equations as a matrix equation.

(ii) Solve this matrix equation. *[6 marks]*

(e) The function f can also be written $f(x) = x(x-1)(rx-s)$ where r and s are integers. Find r and s. *[3 marks]*

3. *[Maximum mark: 14]*

Clara organizes cans in triangular piles, where each row has one less can than the row below. For example, the pile of 15 cans shown has 5 cans in the bottom row and 4 cans in the row above it.



(a) A pile has 20 cans in the bottom row. Show that the pile contains 210 cans. *[4 marks]*

(b) There are 3240 cans in a pile. How many cans are in the bottom row? *[4 marks]*

(c) (i) There are S cans and they are organized in a triangular pile with n cans in the bottom row. Show that $n^2 + n - 2S = 0$.

(ii) Clara has 2100 cans. Explain why she cannot organize them in a triangular pile. *[6 marks]*

4. *[Maximum mark: 21]*

The function f is defined as $f(x) = (2x+1)e^{-x}$, $0 \le x \le 3$. The point $P(0, 1)$ lies on the graph of $f(x)$, and there is a maximum point at Q.

(a) Sketch the graph of $y = f(x)$, labelling the points P and Q. *[3 marks]*

(b) (i) Show that $f'(x) = (1-2x)e^{-x}$.

(ii) Find the **exact** coordinates of Q. *[7 marks]*

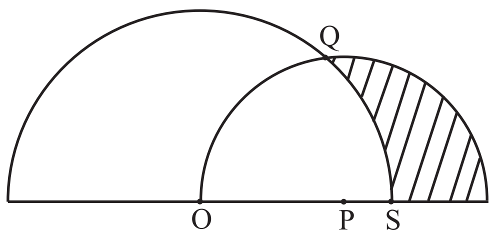
(c) The equation $f(x) = k$, where $k \in \mathbb{R}$, has two solutions. Write down the range of values of k. *[2 marks]*

(d) Given that $f''(x) = e^{-x}(-3+2x)$, show that the curve of f has only one point of inflexion. *[2 marks]*

(e) Let R be the point on the curve of f with x-coordinate 3. Find the area of the region enclosed by the curve and the line (PR). *[7 marks]*

5. *[Maximum mark: 17]*

The following diagram shows two semi-circles. The larger one has centre O and radius 4 cm. The smaller one has centre P, radius 3 cm, and passes through O. The line (OP) meets the larger semi-circle at S. The semi-circles intersect at Q.



(a) (i) Explain why OPQ is an isosceles triangle.

(ii) Use the cosine rule to show that $\cos \mathrm{O\hat{P}Q} = \frac{1}{9}$.

(iii) Hence show that $\sin \mathrm{O\hat{P}Q} = \frac{\sqrt{80}}{9}$.

(iv) Find the area of the triangle OPQ. *[7 marks]*

(b) Consider the smaller semi-circle, with centre P.

(i) Write down the size of $\mathrm{O\hat{P}Q}$.

(ii) Calculate the area of the sector OPQ. *[3 marks]*

(c) Consider the larger semi-circle, with centre O. Calculate the area of the sector QOS. *[3 marks]*

(d) Hence calculate the area of the shaded region. *[4 marks]*