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**Mathematics**  
**Higher level**  
**Paper 3 – sets, relations and groups**

Thursday 21 November 2019 (afternoon)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Let  $A = \{1, 3, 4, 5, 8, 9\}$ ,  $B = \{1, 5, 6, 7, 9\}$  and  $C = \{1, 2, 7, 8, 9\}$ .

- (a) (i) Find  $(A \setminus B) \setminus C$  where  $\setminus$  represents set difference. [5]
- (ii) Find  $A \setminus (B \setminus C)$ .
- (iii) Hence determine whether set difference is associative. [5]
- (b) Find  $(A \Delta B) \Delta C$  where  $\Delta$  represents symmetric difference. [2]
- (c) By considering the sets  $A$ ,  $B$  and  $C$ , determine whether symmetric difference is distributive over intersection. [5]

2. [Maximum mark: 14]

The set  $\{-4, -3, -2, -1, 0, 1, 2, 3\}$  together with the binary operation,  $*$ , forms a group, as defined in the following Cayley table.

*	-4	-3	-2	-1	0	1	2	3
-4	0	1	2	3	-4	-3	-2	-1
-3	1	<i>a</i>	3	-4	-3	-2	-1	0
-2	2	3	-4	-3	-2	-1	0	1
-1	3	-4	-3	-2	-1	0	1	2
0	-4	-3	-2	-1	0	1	2	3
1	-3	<i>b</i>	-1	0	1	2	<i>c</i>	-4
2	-2	-1	0	1	2	3	-4	-3
3	-1	0	1	2	3	-4	-3	-2

- (a) (i) Explain what is meant by the term Latin square. [4]
- (ii) Hence write down the values of  $a$ ,  $b$  and  $c$ .
- (b) (i) Write down the identity element of this group. [2]
- (ii) Hence state the inverse of the element  $-4$ .
- (c) By finding the order of elements, determine whether this group is cyclic. [3]
- (d) Find a subgroup of order 4. [2]

(This question continues on the following page)

**(Question 2 continued)**

There is an isomorphism,  $f$ , from the group  $\{-4, -3, -2, -1, 0, 1, 2, 3\}, *$  to the group  $\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8$  where  $+_8$  is the operation addition modulo 8.

(e) Given that  $f(1) = 1$ , find the value of  $f(-3)$ . [3]

**3. [Maximum mark: 13]**

(a) Let  $V$  be the set of three-dimensional vectors. A relation  $R$  is defined on  $V$  by  $aRb$  if and only if  $a \cdot b = 0$ . Determine with reasons whether  $R$  is

- (i) reflexive;
- (ii) symmetric;
- (iii) transitive. [3]

(b) Let  $W$  be the set of **non-zero** three-dimensional vectors. A relation  $S$  is defined on  $W$  by  $aSb$  if and only if  $a \times b = \mathbf{0}$ . Determine with reasons whether  $S$  is

- (i) reflexive;
- (ii) symmetric;
- (iii) transitive. [5]

(c) (i) Exactly one of  $R$  and  $S$  is an equivalence relation. State which relation this is.

(ii) For this equivalence relation,  $\begin{pmatrix} -2 \\ y \\ -4 \end{pmatrix}$  belongs to the equivalence class

containing  $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ . Find the value of  $y$ . [3]

(d) The relation  $S$  from part (b) is now defined on the set  $V$  from part (a). Determine, with a reason, whether  $S$  is transitive on  $V$ . [2]

4. [Maximum mark: 11]

- (a) Let  $\{G, *\}$  be a group.  
Prove that  $\{G, *\}$  has **exactly** one identity element. [3]
- (b) The binary operation  $\otimes$  is defined on the set of real numbers by  $a \otimes b = a|b|$ .
- (i) Determine whether  $\otimes$  is associative, justifying your answer.
- (ii) Determine whether there is an identity element for  $\otimes$ , justifying your answer. [8]
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