Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – sets, relations and groups

Thursday 21 November 2019 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Let $A = \{1, 3, 4, 5, 8, 9\}$, $B = \{1, 5, 6, 7, 9\}$ and $C = \{1, 2, 7, 8, 9\}$.

(a) (i) Find $(A \setminus B) \setminus C$ where \ represents set difference.

(ii) Find $A \setminus (B \setminus C)$.

(iii) Hence determine whether set difference is associative. [5]

(b) Find $(A \Delta B) \Delta C$ where Δ represents symmetric difference. [2]

(c) By considering the sets A, B and C, determine whether symmetric difference is distributive over intersection. [5]

2. [Maximum mark: 14]

The set $\{-4, -3, -2, -1, 0, 1, 2, 3\}$ together with the binary operation, $*$, forms a group, as defined in the following Cayley table.

$*$	–4	–3	–2	–1	0	1	2	3
–4	0	1	2	3	–4	–3	–2	–1
–3	1	a	3	–4	–3	–2	–1	0
–2	2	3	–4	–3	–2	–1	0	1
–1	3	–4	–3	–2	–1	0	1	2
0	–4	–3	–2	–1	0	1	2	3
1	–3	b	–1	0	1	2	c	–4
2	–2	–1	0	1	2	3	–4	–3
3	–1	0	1	2	3	–4	–3	–2

(a) (i) Explain what is meant by the term Latin square.

(ii) Hence write down the values of a, b and c. [4]

(b) (i) Write down the identity element of this group.

(ii) Hence state the inverse of the element -4. [2]

(c) By finding the order of elements, determine whether this group is cyclic. [3]

(d) Find a subgroup of order 4. [2]

(This question continues on the following page)

(Question 2 continued)

There is an isomorphism, f, from the group $\{\{-4, -3, -2, -1, 0, 1, 2, 3\}, *\}$ to the group $\{\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8\}$ where $+_8$ is the operation addition modulo 8.

(e) Given that $f(1) = 1$, find the value of $f(-3)$. [3]

3. [Maximum mark: 13]

(a) Let V be the set of three-dimensional vectors. A relation R is defined on V by $\boldsymbol{a}R\boldsymbol{b}$ if and only if $\boldsymbol{a} \bullet \boldsymbol{b} = 0$. Determine with reasons whether R is

(i) reflexive;

(ii) symmetric;

(iii) transitive. [3]

(b) Let W be the set of **non-zero** three-dimensional vectors. A relation S is defined on W by $\boldsymbol{a}S\boldsymbol{b}$ if and only if $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$. Determine with reasons whether S is

(i) reflexive;

(ii) symmetric;

(iii) transitive. [5]

(c) (i) Exactly one of R and S is an equivalence relation. State which relation this is.

(ii) For this equivalence relation, $\begin{pmatrix} -2 \\ y \\ -4 \end{pmatrix}$ belongs to the equivalence class containing $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$. Find the value of y. [3]

(d) The relation S from part (b) is now defined on the set V from part (a). Determine, with a reason, whether S is transitive on V. [2]

4. [Maximum mark: 11]

(a) Let $\{G, *\}$ be a group.
Prove that $\{G, *\}$ has **exactly** one identity element. [3]

(b) The binary operation $\otimes$ is defined on the set of real numbers by $a \otimes b = a|b|$.

(i) Determine whether $\otimes$ is associative, justifying your answer.

(ii) Determine whether there is an identity element for $\otimes$, justifying your answer. [8]