Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – statistics and probability

Wednesday 9 May 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 11]

The weights, X kg, of the males of a species of bird may be assumed to be normally distributed with mean 4.8 kg and standard deviation 0.2 kg.

(a) Find the probability that a randomly chosen male bird weighs between 4.75 kg and 4.85 kg. [1]

The weights, Y kg, of female birds of the same species may be assumed to be normally distributed with mean 2.7 kg and standard deviation 0.15 kg.

(b) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird. [6]

(c) Two randomly chosen male birds and three randomly chosen female birds are placed on a weighing machine that has a weight limit of 18 kg. Find the probability that the total weight of these five birds is greater than the weight limit. [4]

2. [Maximum mark: 8]

Consider an unbiased tetrahedral (four-sided) die with faces labelled 1, 2, 3 and 4 respectively.
The random variable X represents the number of throws required to obtain a 1.

(a) State the distribution of X. [1]

(b) Show that the probability generating function, $G(t)$, for X is given by $G(t) = \frac{t}{4 - 3t}$. [4]

(c) Find $G'(t)$. [2]

(d) Determine the mean number of throws required to obtain a 1. [1]

3. [Maximum mark: 12]

A smartphone's battery life is defined as the number of hours a fully charged battery can be used before the smartphone stops working. A company claims that the battery life of a model of smartphone is, on average, 9.5 hours. To test this claim, an experiment is conducted on a random sample of 20 smartphones of this model. For each smartphone, the battery life, b hours, is measured and the sample mean, $\bar{b}$, calculated. It can be assumed the battery lives are normally distributed with standard deviation 0.4 hours.

(a) State suitable hypotheses for a two-tailed test. [1]

(b) Find the critical region for testing $\bar{b}$ at the 5% significance level. [4]

It is then found that this model of smartphone has an average battery life of 9.8 hours.

(c) Find the probability of making a Type II error. [3]

Another model of smartphone whose battery life may be assumed to be normally distributed with mean μ hours and standard deviation 1.2 hours is tested. A researcher measures the battery life of six of these smartphones and calculates a confidence interval of $[10.2, 11.4]$ for μ.

(d) Calculate the confidence level of this interval. [4]

4. [Maximum mark: 11]

The random variables X, Y follow a bivariate normal distribution with product moment correlation coefficient ρ.

(a) State suitable hypotheses to investigate whether or not a negative linear association exists between X and Y. [1]

A random sample of 11 observations on X, Y was obtained and the value of the sample product moment correlation coefficient, r, was calculated to be -0.708.

(b) (i) Determine the p-value.

(ii) State your conclusion at the 1% significance level. [4]

The covariance of the random variables U, V is defined by

$$\mathrm{Cov}(U, V) = \mathrm{E}\big((U - \mathrm{E}(U))(V - \mathrm{E}(V))\big).$$

(c) (i) Show that $\mathrm{Cov}(U, V) = \mathrm{E}(UV) - \mathrm{E}(U)\mathrm{E}(V)$.

(ii) Hence show that if U, V are independent random variables then the population product moment correlation coefficient, ρ, is zero. [6]

5. [Maximum mark: 8]

The random variable X has a binomial distribution with parameters n and p.

(a) Show that $P = \frac{X}{n}$ is an unbiased estimator of p. [2]

Let $U = nP(1 - P)$.

(b) (i) Show that $\mathrm{E}(U) = (n - 1)p(1 - p)$.

(ii) Hence write down an unbiased estimator of $\mathrm{Var}(X)$. [6]