Mathematics
Higher level
Paper 3 – calculus

Wednesday 9 May 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2218 – 7208

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 10]

(a) Given that $n > \ln n$ for $n > 0$, use the comparison test to show that the series $\sum_{n=0}^{\infty} \frac{1}{\ln(n+2)}$ is divergent. [3]

(b) Find the interval of convergence for $\sum_{n=0}^{\infty} \frac{(3x)^n}{\ln(n+2)}$. [7]

2. [Maximum mark: 6]

The function f is defined by

$$f(x) = \begin{cases} |x-2|+1 & x < 2 \\ ax^2 + bx & x \geq 2 \end{cases}$$

where a and b are real constants.

Given that both f and its derivative are continuous at $x = 2$, find the value of a and the value of b.

3. [Maximum mark: 11]

(a) Find the value of $\int_4^{\infty} \frac{1}{x^3}\,\mathrm{d}x$. [3]

(b) Illustrate graphically the inequality $\sum_{n=5}^{\infty} \frac{1}{n^3} < \int_4^{\infty} \frac{1}{x^3}\,\mathrm{d}x < \sum_{n=4}^{\infty} \frac{1}{n^3}$. [4]

(c) Hence write down a lower bound for $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [1]

(d) Find an upper bound for $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [3]

4. [Maximum mark: 11]

The function f is defined by $f(x) = (\arcsin x)^2$, $-1 \le x \le 1$.

(a) Show that $f'(0) = 0$. [2]

The function f satisfies the equation $\left(1 - x^2\right) f''(x) - xf'(x) - 2 = 0$.

(b) By differentiating the above equation twice, show that

$$\left(1 - x^2\right) f^{(4)}(x) - 5xf^{(3)}(x) - 4f''(x) = 0$$

where $f^{(3)}(x)$ and $f^{(4)}(x)$ denote the 3rd and 4th derivative of $f(x)$ respectively. [4]

(c) Hence show that the Maclaurin series for $f(x)$ up to and including the term in x^4 is $x^2 + \frac{1}{3}x^4$. [3]

(d) Use this series approximation for $f(x)$ with $x = \frac{1}{2}$ to find an approximate value for π^2. [2]

5. [Maximum mark: 12]

Consider the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^p + 1$ where $x \in \mathbb{R}, x \ne 0$ and p is a positive integer, $p > 1$.

(a) Solve the differential equation given that $y = -1$ when $x = 1$. Give your answer in the form $y = f(x)$. [8]

(b) (i) Show that the x-coordinate(s) of the points on the curve $y = f(x)$ where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ satisfy the equation $x^{p-1} = \frac{1}{p}$.

(ii) Deduce the set of values for p such that there are two points on the curve $y = f(x)$ where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Give a reason for your answer. [4]