Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2218–7206

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the complex number $z = \frac{2 + 7\mathrm{i}}{6 + 2\mathrm{i}}$.

(a) Express z in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{Q}$. [2]

(b) Find the exact value of the modulus of z. [2]

(c) Find the argument of z, giving your answer to 4 decimal places. [2]

2. [Maximum mark: 5]

The polynomial $x^4 + px^3 + qx^2 + rx + 6$ is exactly divisible by each of $(x-1)$, $(x-2)$ and $(x-3)$.

Find the values of p, q and r.

3. [Maximum mark: 6]

The random variable X has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

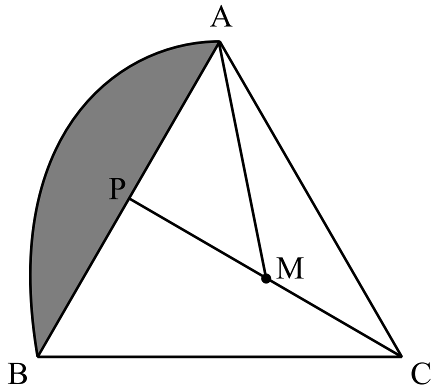
(a) Sketch the probability density function for X, and shade the region representing $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(b) Find the value of $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(c) Find the value of k for which $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5$. [2]

4. [Maximum mark: 8]

Consider the following diagram.



The sides of the equilateral triangle ABC have lengths $1\,\text{m}$. The midpoint of [AB] is denoted by P. The circular arc AB has centre, M, the midpoint of [CP].

(a) (i) Find AM.

(ii) Find $\hat{\text{AMP}}$ in radians. [5]

(b) Find the area of the shaded region. [3]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

5. [Maximum mark: 6]

(a) Express the binomial coefficient $\binom{3n+1}{3n-2}$ as a polynomial in n. [3]

(b) Hence find the least value of n for which $\binom{3n+1}{3n-2} > 10^6$. [3]

6. [Maximum mark: 7]

Use mathematical induction to prove that $(1-a)^n > 1 - na$ for $\{n : n \in \mathbb{Z}^+, n \geq 2\}$ where $0 < a < 1$.

7. [Maximum mark: 5]

A point P moves in a straight line with velocity v ms^{-1} given by $v(t) = e^{-t} - 8t^2e^{-2t}$ at time t seconds, where $t \geq 0$.

(a) Determine the first time t_1 at which P has zero velocity. [2]

(b) (i) Find an expression for the acceleration of P at time t.

(ii) Find the value of the acceleration of P at time t_1. [3]

8. [Maximum mark: 7]

The random variable X has a binomial distribution with parameters n and p.
It is given that $\mathrm{E}(X) = 3.5$.

(a) Find the least possible value of n. [2]

It is further given that $\mathrm{P}(X \leq 1) = 0.09478$ correct to 4 significant figures.

(b) Determine the value of n and the value of p. [5]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 13]

The number of taxis arriving at Cardiff Central railway station can be modelled by a Poisson distribution. During busy periods of the day, taxis arrive at a mean rate of 5.3 taxis every 10 minutes. Let T represent a random 10 minute busy period.

(a) (i) Find the probability that exactly 4 taxis arrive during T.

(ii) Find the most likely number of taxis that would arrive during T.

(iii) Given that more than 5 taxis arrive during T, find the probability that exactly 7 taxis arrive during T. [7]

During quiet periods of the day, taxis arrive at a mean rate of 1.3 taxis every 10 minutes.

(b) Find the probability that during a period of 15 minutes, of which the first 10 minutes is busy and the next 5 minutes is quiet, that exactly 2 taxis arrive. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Consider the expression $f(x) = \tan\left(x + \frac{\pi}{4}\right)\cot\left(\frac{\pi}{4} - x\right)$.

(a) (i) Sketch the graph of $y = f(x)$ for $-\frac{5\pi}{8} \le x \le \frac{\pi}{8}$.

(ii) With reference to your graph, explain why f is a function on the given domain.

(iii) Explain why f has no inverse on the given domain.

(iv) Explain why f is not a function for $-\frac{3\pi}{4} \le x \le \frac{\pi}{4}$. [5]

The expression $f(x)$ can be written as $g(t)$ where $t = \tan x$.

(b) Show that $g(t) = \left(\frac{1+t}{1-t}\right)^2$. [3]

(c) Sketch the graph of $y = g(t)$ for $t \le 0$. Give the coordinates of any intercepts and the equations of any asymptotes. [3]

(d) Let α, β be the roots of $g(t) = k$, where $0 < k < 1$.

(i) Find α and β in terms of k.

(ii) Show that $\alpha + \beta < -2$. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 19]

A curve C is given by the implicit equation $x + y - \cos(xy) = 0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{1 + y\sin(xy)}{1 + x\sin(xy)}\right)$. [5]

(b) The curve $xy = -\frac{\pi}{2}$ intersects C at P and Q.

(i) Find the coordinates of P and Q.

(ii) Given that the gradients of the tangents to C at P and Q are m_1 and m_2 respectively, show that $m_1 \times m_2 = 1$. [7]

(c) Find the coordinates of the three points on C, nearest the origin, where the tangent is parallel to the line $y = -x$. [7]