Mathematics
Higher level
Paper 3 – sets, relations and groups

Thursday 16 November 2017 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 11]

Consider the group $\{G, \times_{18}\}$ defined on the set $\{1, 5, 7, 11, 13, 17\}$ where $\times_{18}$ denotes multiplication modulo 18. The group $\{G, \times_{18}\}$ is shown in the following Cayley table.

$\times_{18}$	**1**	**5**	**7**	**11**	**13**	**17**
1	1	5	7	11	13	17
5	5	7	17	1	11	13
7	7	17	13	5	1	11
11	11	1	5	13	17	7
13	13	11	1	17	7	5
17	17	13	11	7	5	1

(a) (i) Find the order of elements $5, 7$ and 17 in $\{G, \times_{18}\}$.

(ii) State whether or not $\{G, \times_{18}\}$ is cyclic, justifying your answer. [6]

The subgroup of $\{G, \times_{18}\}$ of order two is denoted by $\{K, \times_{18}\}$.

(b) Write down the elements in set K. [1]

(c) Find the left cosets of K in $\{G, \times_{18}\}$. [4]

2. [Maximum mark: 8]

A, B and C are three subsets of a universal set.

(a) Represent each of the following sets on a Venn diagram,

(i) $A \Delta B$, the symmetric difference of the sets A and B;

(ii) $A \cap (B \cup C)$. [2]

Consider the sets $P = \{1, 2, 3\}$, $Q = \{2, 3, 4\}$ and $R = \{1, 3, 5\}$.

(b) (i) For sets P, Q and R, verify that $P \cup (Q \Delta R) \neq (P \cup Q) \Delta (P \cup R)$.

(ii) In the context of the distributive law, describe what the result in part (b)(i) illustrates. [6]

3. [Maximum mark: 9]

The relation R is defined on $\mathbb{R} \times \mathbb{R}$ such that $(x_1, y_1)R(x_2, y_2)$ if and only if $x_1 y_1 = x_2 y_2$.

(a) Show that R is an equivalence relation. [5]

(b) Determine the equivalence class of R containing the element $(1, 2)$ and illustrate this graphically. [4]

4. [Maximum mark: 14]

The set S is defined as the set of real numbers greater than 1.
The binary operation $*$ is defined on S by $x * y = (x-1)(y-1)+1$ for all $x, y \in S$.

(a) Show that $x * y \in S$ for all $x, y \in S$. [2]

(b) Show that the operation $*$ on the set S is

(i) commutative;

(ii) associative. [7]

(c) Show that 2 is the identity element. [2]

Let $a \in S$.

(d) Show that each element $a \in S$ has an inverse. [3]

5. [Maximum mark: 8]

Let $f: G \to H$ be a homomorphism between groups $\{G, *\}$ and $\{H, \circ\}$ with identities e_G and e_H respectively.

(a) Prove that $f(e_G) = e_H$. [2]

(b) Prove that $\mathrm{Ker}(f)$ is a subgroup of $\{G, *\}$. [6]