# Mathematics
# Higher level
# Paper 2

Friday 5 May 2017 (morning)

Candidate session number

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2 hours

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.



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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

**1.** [Maximum mark: 4]

There are $75$ players in a golf club who take part in a golf tournament. The scores obtained on the $18\text{th}$ hole are as shown in the following table.

| Score | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Frequency | 3 | 15 | 28 | 17 | 9 | 3 |

(a) One of the players is chosen at random. Find the probability that this player's score was $5$ or more. [2]

(b) Calculate the mean score. [2]

**2.** [Maximum mark: 9]

Consider the curve defined by the equation $4x^2 + y^2 = 7$.

(a) Find the equation of the normal to the curve at the point $\left(1, \sqrt{3}\right)$. [6]

(b) Find the volume of the solid formed when the region bounded by the curve, the $x$-axis for $x \geq 0$ and the $y$-axis for $y \geq 0$ is rotated through $2\pi$ about the $x$-axis. [3]

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**3.** [Maximum mark: 7]

Packets of biscuits are produced by a machine. The weights $X$, in grams, of packets of biscuits can be modelled by a normal distribution where $X \sim \mathrm{N}(\mu, \sigma^2)$. A packet of biscuits is considered to be underweight if it weighs less than $250$ grams.

(a) Given that $\mu = 253$ and $\sigma = 1.5$ find the probability that a randomly chosen packet of biscuits is underweight. [2]

The manufacturer makes the decision that the probability that a packet is underweight should be $0.002$. To do this $\mu$ is increased and $\sigma$ remains unchanged.

(b) Calculate the new value of $\mu$ giving your answer correct to two decimal places. [3]

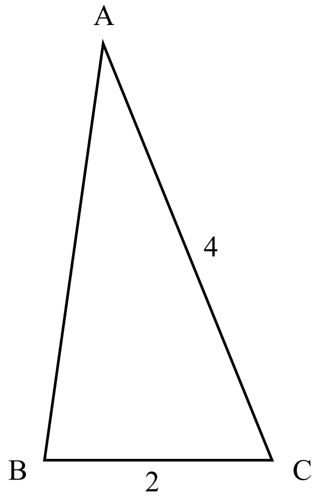
The manufacturer is happy with the decision that the probability that a packet is underweight should be $0.002$, but is unhappy with the way in which this was achieved. The machine is now adjusted to reduce $\sigma$ and return $\mu$ to $253$.

(c) Calculate the new value of $\sigma$. [2]

**4.** [Maximum mark: 6]

(a) Find the set of values of $k$ that satisfy the inequality $k^2 - k - 12 < 0$. [2]

(b) The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB. [4]



**5.** [Maximum mark: 9]

John likes to go sailing every day in July. To help him make a decision on whether it is safe to go sailing he classifies each day in July as windy or calm. Given that a day in July is calm, the probability that the next day is calm is $0.9$. Given that a day in July is windy, the probability that the next day is calm is $0.3$. The weather forecast for the 1st July predicts that the probability that it will be calm is $0.8$.

(a) Draw a tree diagram to represent this information for the first three days of July. [3]

(b) Find the probability that the 3rd July is calm. [2]

(c) Find the probability that the 1st July was calm given that the 3rd July is windy. [4]

**6.** [Maximum mark: 5]

Given that $\log_{10}\left(\frac{1}{2\sqrt{2}}(p+2q)\right) = \frac{1}{2}\left(\log_{10} p + \log_{10} q\right)$, $p > 0$, $q > 0$, find $p$ in terms of $q$.

**7.** [Maximum mark: 4]

Given that $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{c} \neq \boldsymbol{0}$ prove that $\boldsymbol{a} + \boldsymbol{c} = s\boldsymbol{b}$ where $s$ is a scalar.

**8.** [Maximum mark: 6]

In a trial examination session a candidate at a school has to take $18$ examination papers including the physics paper, the chemistry paper and the biology paper. No two of these three papers may be taken consecutively. There is no restriction on the order in which the other examination papers may be taken.

Find the number of different orders in which these $18$ examination papers may be taken.

Do **not** write solutions on this page.

## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

**9.** [Maximum mark: 22]

The points A, B and C have the following position vectors with respect to an origin O.

$$\vec{\mathrm{OA}} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$$

$$\vec{\mathrm{OB}} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$$

$$\vec{\mathrm{OC}} = \boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k}$$

(a) Find the vector equation of the line (BC). [3]

(b) Determine whether or not the lines (OA) and (BC) intersect. [6]

(c) Find the Cartesian equation of the plane $\Pi_1$, which passes through C and is perpendicular to $\vec{\mathrm{OA}}$. [3]

(d) Show that the line (BC) lies in the plane $\Pi_1$. [2]

The plane $\Pi_2$ contains the points O, A and B and the plane $\Pi_3$ contains the points O, A and C.

(e) Verify that $2\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to the plane $\Pi_2$. [3]

(f) Find a vector perpendicular to the plane $\Pi_3$. [1]

(g) Find the acute angle between the planes $\Pi_2$ and $\Pi_3$. [4]

**Turn over**

Do **not** write solutions on this page.

**10.** [Maximum mark: 15]

A continuous random variable $X$ has probability density function $f$ given by

$$f(x) = \begin{cases} \dfrac{x^2}{a} + b, & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$ where $a$ and $b$ are positive constants.

It is given that $\mathrm{P}(X \ge 2) = 0.75$.

(a) Show that $a = 32$ and $b = \dfrac{1}{12}$. [5]

(b) Find $\mathrm{E}(X)$. [2]

(c) Find $\mathrm{Var}(X)$. [2]

(d) Find the median of $X$. [3]

Eight independent observations of $X$ are now taken and the random variable $Y$ is the number of observations such that $X \ge 2$.

(e) Find $\mathrm{E}(Y)$. [2]

(f) Find $\mathrm{P}(Y \ge 3)$. [1]

Do **not** write solutions on this page.

**11.** [Maximum mark: 13]

It is given that $f(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$ where $a$ and $b$ are positive integers.

(a) Given that $x^2 - 1$ is a factor of $f(x)$ find the value of $a$ and the value of $b$. [4]

(b) Factorize $f(x)$ into a product of linear factors. [3]

(c) Sketch the graph of $y = f(x)$, labelling the maximum and minimum points and the $x$ and $y$ intercepts. [3]

(d) Using your graph state the range of values of $c$ for which $f(x) = c$ has exactly two distinct real roots. [3]

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