Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Thursday 10 November 2016 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Find the coordinates of the point of intersection of the planes defined by the equations $x+y+z=3$, $x-y+z=5$ and $x+y+2z=6$.

2. [Maximum mark: 4]

The faces of a fair six-sided die are numbered $1, 2, 2, 4, 4, 6$. Let X be the discrete random variable that models the score obtained when this die is rolled.

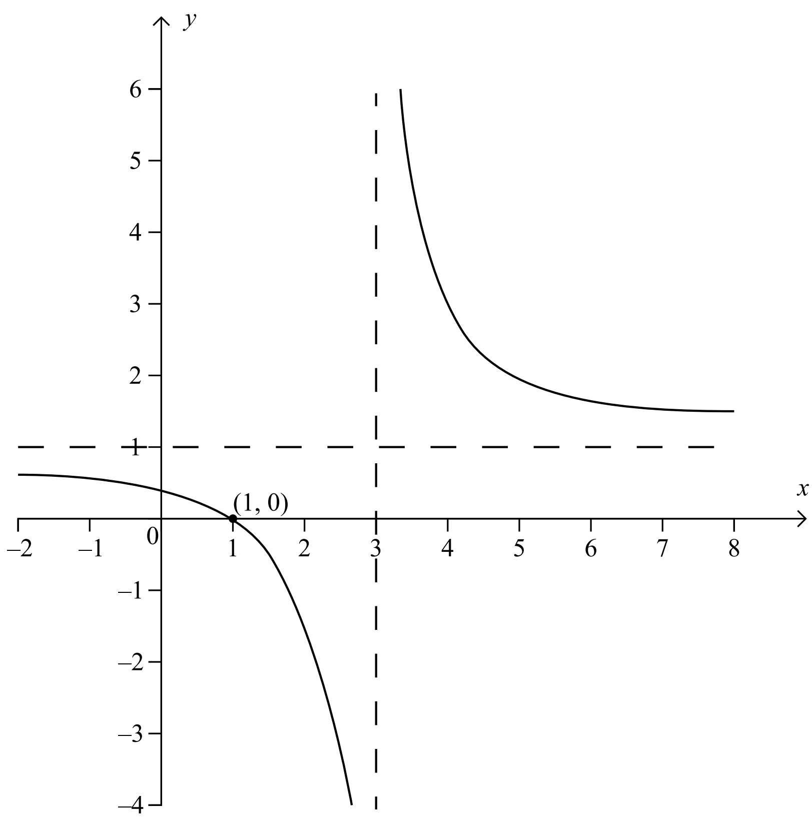
(a) Complete the probability distribution table for X. [2]

x				
$\mathrm{P}(X = x)$				

(b) Find the expected value of X. [2]

3. [Maximum mark: 4]

A rational function is defined by $f(x) = a + \dfrac{b}{x - c}$ where the parameters $a, b, c \in \mathbb{Z}$ and $x \in \mathbb{R} \setminus \{c\}$. The following diagram represents the graph of $y = f(x)$.



Using the information on the graph,

(a) state the value of a and the value of c; [2]

(b) find the value of b. [2]

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4. [Maximum mark: 5]

Consider the vectors $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 2\boldsymbol{k}$.

(a) Find $\boldsymbol{a} \times \boldsymbol{b}$. [2]

(b) Hence find the Cartesian equation of the plane containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and passing through the point $(1, 0, -1)$. [3]

5. [Maximum mark: 6]

The quadratic equation $x^2 - 2kx + (k - 1) = 0$ has roots α and β such that $\alpha^2 + \beta^2 = 4$. Without solving the equation, find the possible values of the real number k.

6. [Maximum mark: 7]

The sum of the first n terms of a sequence $\{u_n\}$ is given by $S_n = 3n^2 - 2n$, where $n \in \mathbb{Z}^+$.

(a) Write down the value of u_1. [1]

(b) Find the value of u_6. [2]

(c) Prove that $\{u_n\}$ is an arithmetic sequence, stating clearly its common difference. [4]

7. [Maximum mark: 5]

Solve the equation $4^x + 2^{x+2} = 3$.

8. [Maximum mark: 6]

Consider the lines l_1 and l_2 defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \text{ and } l_2 : \frac{6 - x}{3} = \frac{y - 2}{4} = 1 - z \text{ where } a \text{ is a constant.}$$

Given that the lines l_1 and l_2 intersect at a point P,

(a) find the value of a; [4]

(b) determine the coordinates of the point of intersection P. [2]

9. [Maximum mark: 9]

A curve has equation $3x - 2y^2 e^{x-1} = 2$.

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [5]

(b) Find the equations of the tangents to this curve at the points where the curve intersects the line $x = 1$. [4]

10. [Maximum mark: 9]

Consider two events A and B defined in the same sample space.

(a) Show that $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$. [3]

(b) Given that $\mathrm{P}(A \cup B) = \frac{4}{9}$, $\mathrm{P}(B \mid A) = \frac{1}{3}$ and $\mathrm{P}(B \mid A') = \frac{1}{6}$,

(i) show that $\mathrm{P}(A) = \frac{1}{3}$;

(ii) hence find $\mathrm{P}(B)$. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 22]

Let $y = e^x \sin x$.

(a) Find an expression for $\frac{dy}{dx}$. [2]

(b) Show that $\frac{d^2y}{dx^2} = 2e^x \cos x$. [2]

Consider the function f defined by $f(x) = e^x \sin x$, $0 \le x \le \pi$.

(c) Show that the function f has a local maximum value when $x = \frac{3\pi}{4}$. [2]

(d) Find the x-coordinate of the point of inflexion of the graph of f. [2]

(e) Sketch the graph of f, clearly indicating the position of the local maximum point, the point of inflexion and the axes intercepts. [3]

(f) Find the area of the region enclosed by the graph of f and the x-axis. [6]

The curvature at any point (x, y) on a graph is defined as $\kappa = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}$.

(g) Find the value of the curvature of the graph of f at the local maximum point. [3]

(h) Find the value κ for $x = \frac{\pi}{2}$ and comment on its meaning with respect to the shape of the graph. [2]

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12. [Maximum mark: 19]

Let ω be one of the non-real solutions of the equation $z^3 = 1$.

(a) Determine the value of

(i) $1 + \omega + \omega^2$;

(ii) $1 + \omega^* + (\omega^*)^2$. [4]

(b) Show that $(\omega - 3\omega^2)(\omega^2 - 3\omega) = 13$. [4]

Consider the complex numbers $p = 1 - 3\mathrm{i}$ and $q = x + (2x + 1)\mathrm{i}$, where $x \in \mathbb{R}$.

(c) Find the values of x that satisfy the equation $|p| = |q|$. [5]

(d) Solve the inequality $\mathrm{Re}(pq) + 8 < (\mathrm{Im}(pq))^2$. [6]

13. [Maximum mark: 19]

(a) Find the value of $\sin\frac{\pi}{4} + \sin\frac{3\pi}{4} + \sin\frac{5\pi}{4} + \sin\frac{7\pi}{4} + \sin\frac{9\pi}{4}$. [2]

(b) Show that $\frac{1 - \cos 2x}{2\sin x} \equiv \sin x, x \neq k\pi$ where $k \in \mathbb{Z}$. [2]

(c) Use the principle of mathematical induction to prove that

$\sin x + \sin 3x + \ldots + \sin(2n - 1)x = \frac{1 - \cos 2nx}{2\sin x}, n \in \mathbb{Z}^+, x \neq k\pi$ where $k \in \mathbb{Z}$. [9]

(d) Hence or otherwise solve the equation $\sin x + \sin 3x = \cos x$ in the interval $0 < x < \pi$. [6]

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