

**Mathematics**  
**Higher level**  
**Paper 3 – sets, relations and groups**

Wednesday 18 November 2015 (afternoon)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Given the sets  $A$  and  $B$ , use the properties of sets to prove that  $A \cup (B' \cup A)' = A \cup B$ , justifying each step of the proof.

2. [Maximum mark: 14]

The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f: x \rightarrow \begin{cases} 1 & , x \geq 0 \\ -1 & , x < 0 \end{cases}$ .

(a) Prove that  $f$  is

(i) not injective;

(ii) not surjective.

[4]

The relation  $R$  is defined for  $a, b \in \mathbb{R}$  so that  $aRb$  if and only if  $f(a) \times f(b) = 1$ .

(b) Show that  $R$  is an equivalence relation.

[8]

(c) State the equivalence classes of  $R$ .

[2]

3. [Maximum mark: 10]

The set of all permutations of the elements  $1, 2, \dots, 10$  is denoted by  $H$  and the binary operation  $\circ$  represents the composition of permutations.

The permutation  $p = (1\ 2\ 3\ 4\ 5\ 6)(7\ 8\ 9\ 10)$  generates the subgroup  $\{G, \circ\}$  of the group  $\{H, \circ\}$ .

(a) Find the order of  $\{G, \circ\}$ . [2]

(b) State the identity element in  $\{G, \circ\}$ . [1]

(c) Find

(i)  $p \circ p$ ;

(ii) the inverse of  $p \circ p$ . [4]

(d) (i) Find the maximum possible order of an element in  $\{H, \circ\}$ .

(ii) Give an example of an element with this order. [3]

4. [Maximum mark: 18]

The binary operation  $*$  is defined on the set  $T = \{0, 2, 3, 4, 5, 6\}$  by  $a * b = (a + b - ab) \pmod{7}$ ,  $a, b \in T$ .

(a) Copy and complete the following Cayley table for  $\{T, *\}$ . [4]

*	0	2	3	4	5	6
0	0	2	3	4	5	6
2	2	0	6	5	4	3
3	3	6				
4	4	5				
5	5	4				
6	6	3				

(b) Prove that  $\{T, *\}$  forms an Abelian group. [7]

(c) Find the order of each element in  $T$ . [4]

(d) Given that  $\{H, *\}$  is the subgroup of  $\{T, *\}$  of order 2, partition  $T$  into the left cosets with respect to  $H$ . [3]

5. [Maximum mark: 13]

A group  $\{D, \times_3\}$  is defined so that  $D = \{1, 2\}$  and  $\times_3$  is multiplication modulo 3.

A function  $f: \mathbb{Z} \rightarrow D$  is defined as  $f: x \mapsto \begin{cases} 1, & x \text{ is even} \\ 2, & x \text{ is odd} \end{cases}$ .

- (a) Prove that the function  $f$  is a homomorphism from the group  $\{\mathbb{Z}, +\}$  to  $\{D, \times_3\}$ . [6]
  - (b) Find the kernel of  $f$ . [3]
  - (c) Prove that  $\{\text{Ker}(f), +\}$  is a subgroup of  $\{\mathbb{Z}, +\}$ . [4]
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