N15/5/MATHL/HP1/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2015

Mathematics

Higher level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document "**Mathematics HL: Guidance for e-marking November 2015**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**).*
*A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(= 10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. arc length $= \frac{2}{x} = rx \left(\Rightarrow r = \frac{2}{x^2} \right)$ ***M1***

$16 = \frac{1}{2}\left(\frac{2}{x^2}\right)^2 x \left(\Rightarrow \frac{2}{x^3} = 16 \right)$ ***M1***

Note: Award M1s for attempts at the use of arc-length and sector-area formulae.

$x = \frac{1}{2}$ ***A1***

arc length $= 4(\text{cm})$ ***A1***

[4 marks]

2. attempt to integrate one factor and differentiate the other, leading to a sum of two terms ***M1***

$\int x \sin x \, dx = x(-\cos x) + \int \cos x \, dx$ ***(A1)(A1)***

$= -x \cos x + \sin x + c$ ***A1***

Note: Only award final ***A1*** if $+ c$ is seen.

[4 marks]

3. (a) $(2+x)^4 = 2^4 + 4 \cdot 2^3 x + 6 \cdot 2^2 x^2 + 4 \cdot 2x^3 + x^4$ ***M1(A1)***

Note: Award ***M1*** for an expansion, by whatever method, giving five terms in any order.

$= 16 + 32x + 24x^2 + 8x^3 + x^4$ ***A1***

Note: Award ***M1A1A0*** for correct expansion not given in ascending powers of x.

[3 marks]

(b) let $x = 0.1$ (in the binomial expansion) ***(M1)***

$2.1^4 = 16 + 3.2 + 0.24 + 0.008 + 0.0001$ ***(A1)***

$= 19.4481$ ***A1***

Note: At most one of the marks can be implied.

[3 marks]

Total [6 marks]

4. (a) $\frac{dy}{dx} = (1-x)^{-2} \left(= \frac{1}{(1-x)^2} \right)$ ***(M1)A1***

[2 marks]

continued…

Question 4 continued

(b) gradient of Tangent $= \frac{1}{4}$ ***(A1)***

gradient of Normal $= -4$ ***(M1)***

$y + \frac{1}{2} = -4(x-3)$ or attempt to find c in $y = mx + c$ ***M1***

$8x + 2y - 23 = 0$ ***A1***

[4 marks]

Total [6 marks]

5. METHOD 1

$\int_{e}^{e^2} \frac{dx}{x \ln x} = [\ln(\ln x)]_{e}^{e^2}$ ***(M1)A1***

$= \ln(\ln e^2) - \ln(\ln e)$ $(= \ln 2 - \ln 1)$ ***(A1)***

$= \ln 2$ ***A1***

[4 marks]

METHOD 2

$u = \ln x, \frac{du}{dx} = \frac{1}{x}$ ***M1***

$= \int_1^2 \frac{du}{u}$ ***A1***

$= [\ln u]_1^2$ or equivalent in x $(= \ln 2 - \ln 1)$ ***(A1)***

$= \ln 2$ ***A1***

[4 marks]

6. (a) probability that Darren wins $P(W) + P(RRW) + P(RRRRW)$ ***(M1)***

Note: Only award ***M1*** if three terms are seen or are implied by the following numerical equivalent.

Note: Accept equivalent tree diagram for method mark.

$= \frac{2}{6} + \frac{4}{6}\cdot\frac{3}{5}\cdot\frac{2}{4} + \frac{4}{6}\cdot\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}\cdot\frac{2}{2} \quad \left(= \frac{1}{3} + \frac{1}{5} + \frac{1}{15}\right)$ ***A2***

Note: ***A1*** for two correct.

$= \frac{3}{5}$ ***A1***

[4 marks]

continued…

Question 6 continued

(b) **METHOD 1**

the probability that Darren wins is given by
$\mathrm{P}(W) + \mathrm{P}(RRW) + \mathrm{P}(RRRRW) + \ldots$ ***(M1)***

Note: Accept equivalent tree diagram with correctly indicated path for method mark.

$\mathrm{P}(\text{Darren Win}) = \frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \ldots$

or $= \frac{1}{3}\left(1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \ldots\right)$ ***A1***

$= \frac{1}{3}\left(\frac{1}{1-\frac{4}{9}}\right)$ ***A1***

$= \frac{3}{5}$ ***AG***

[3 marks]

METHOD 2

$\mathrm{P}(\text{Darren wins}) = \mathrm{P}$

$\mathrm{P} = \frac{1}{3} + \frac{4}{9}\mathrm{P}$ ***M1A2***

$\frac{5}{9}\mathrm{P} = \frac{1}{3}$

$\mathrm{P} = \frac{3}{5}$ ***AG***

[3 marks]

Total [7 marks]

7. (a) $x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ ***M1A1***

a horizontal tangent occurs if $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ so $y = 0$ ***M1***

we can see from the equation of the curve that this solution is not possible $(0 = 4)$ and so there is not a horizontal tangent ***R1***

[4 marks]

continued…

Question 7 continued

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{2y - x}$ or equivalent with $\frac{\mathrm{d}x}{\mathrm{d}y}$

the tangent is vertical when $2y = x$ ***M1***

substitute into the equation to give $2y^2 = y^2 + 4$ ***M1***

$y = \pm 2$ ***A1***

coordinates are $(4, 2), (-4, -2)$ ***A1***

[4 marks]

Total [8 marks]

8. (a) $\sin\left(\theta + \frac{\pi}{2}\right) = \sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2}$ ***M1***

$= \cos\theta$ ***AG***

Note: Accept a transformation/graphical based approach.

[1 mark]

(b) consider $n = 1$, $f'(x) = a\cos(ax)$ ***M1***

since $\sin\left(ax + \frac{\pi}{2}\right) = \cos ax$ then the proposition is true for $n = 1$ ***R1***

assume that the proposition is true for $n = k$ so $f^{(k)}(x) = a^k \sin\left(ax + \frac{k\pi}{2}\right)$ ***M1***

$f^{(k+1)}(x) = \frac{\mathrm{d}\left(f^{(k)}(x)\right)}{\mathrm{d}x}\left(= a\left(a^k\cos\left(ax + \frac{k\pi}{2}\right)\right)\right)$ ***M1***

$= a^{k+1}\sin\left(ax + \frac{k\pi}{2} + \frac{\pi}{2}\right)$ (using part (a)) ***A1***

$= a^{k+1}\sin\left(ax + \frac{(k+1)\pi}{2}\right)$ ***A1***

given that the proposition is true for $n = k$ then we have shown that the proposition is true for $n = k + 1$. Since we have shown that the proposition is true for $n = 1$ then the proposition is true for all $n \in \mathbb{Z}^+$ ***R1***

Note: Award final ***R1*** only if all prior M and R marks have been awarded.

[7 marks]

Total [8 marks]

9. $(\sin 2x - \sin x) - (\cos 2x - \cos x) = 1$

attempt to use both double-angle formulae, in whatever form ***M1***

$(2\sin x \cos x - \sin x) - (2\cos^2 x - 1 - \cos x) = 1$

or $(2\sin x \cos x - \sin x) - \left(2\cos^2 x - \cos x\right) = 0$ for example ***A1***

Note: Allow any rearrangement of the above equations.

$\sin x(2\cos x - 1) - \cos x(2\cos x - 1) = 0$

$(\sin x - \cos x)\ (2\cos x - 1) = 0$ ***(M1)***

$\tan x = 1$ and $\cos x = \frac{1}{2}$ ***A1A1***

Note: These ***A*** marks are dependent on the ***M*** mark awarded for factorisation.

$x = -\frac{3\pi}{4}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{4}$ ***A2***

Note: Award ***A1*** for two correct answers, which could be for both tan or both cos solutions, for example.

[7 marks]

10. (a) the sum of the roots of the polynomial $= \frac{63}{16}$ ***(A1)***

$$2\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right) = \frac{63}{16}$$ ***M1A1***

Note: The formula for the sum of a geometric sequence must be equated to a value for the ***M1*** to be awarded.

$$1-\left(\frac{1}{2}\right)^n = \frac{63}{64} \Rightarrow \left(\frac{1}{2}\right)^n = \frac{1}{64}$$

$n = 6$ ***A1***

[4 marks]

(b) $\frac{a_0}{a_n} = 2\times 1\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{8}\times\frac{1}{16}, \left(a_n = 16\right)$ ***M1***

$a_0 = 16\times 2\times 1\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{8}\times\frac{1}{16}$

$a_0 = 2^{-5}\left(=\frac{1}{32}\right)$ ***A1***

[2 marks]

Total [6 marks]

Section B

11. (a) $z^3 = 8\left(\cos\left(\frac{\pi}{2} + 2\pi k\right) + \mathrm{i}\sin\left(\frac{\pi}{2} + 2\pi k\right)\right)$ ***(A1)***

attempt the use of De Moivre's Theorem in reverse ***M1***

$z = 2\left(\cos\left(\frac{\pi}{6}\right) + \mathrm{i}\sin\left(\frac{\pi}{6}\right)\right);\ 2\left(\cos\left(\frac{5\pi}{6}\right) + \mathrm{i}\sin\left(\frac{5\pi}{6}\right)\right);$

$2\left(\cos\left(\frac{9\pi}{6}\right) + \mathrm{i}\sin\left(\frac{9\pi}{6}\right)\right)$ ***A2***

Note: Accept cis form.

$z = \pm\sqrt{3} + \mathrm{i},\ -2\mathrm{i}$ ***A2***

Note: Award ***A1*** for two correct solutions in each of the two lines above.

[6 marks]

(b) (i) $z_1 = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + \mathrm{i}\sin\left(\frac{\pi}{4}\right)\right)$ ***A1A1***

(ii) $\left(z_2 = \left(\sqrt{3} + \mathrm{i}\right)\right)$

$z_1 z_2 = (1 + \mathrm{i})\left(\sqrt{3} + \mathrm{i}\right)$ ***M1***

$= \left(\sqrt{3} - 1\right) + \mathrm{i}\left(1 + \sqrt{3}\right)$ ***A1***

(iii) $z_1 z_2 = 2\sqrt{2}\left(\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) + \mathrm{i}\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)\right)$ ***M1A1***

$\tan\frac{5\pi}{12} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ ***A1***

$= 2 + \sqrt{3}$ ***M1A1***

Note: Award final ***M1*** for an attempt to rationalise the fraction.

(iv) $z_2{}^p = 2^p\left(\operatorname{cis}\left(\frac{p\pi}{6}\right)\right)$ ***(M1)***

$z_2{}^p$ is a positive real number when $p = 12$ ***A1***

[11 marks]

Total [17 marks]

12. (a) $f(-x) = (-x)\sqrt{1-(-x)^2}$ *M1*

$= -x\sqrt{1-x^2}$

$= -f(x)$ *R1*

hence f is odd *AG*

[2 marks]

(b) $f'(x) = x \cdot \frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}} \cdot -2x + \left(1-x^2\right)^{\frac{1}{2}}$ *M1A1A1*

[3 marks]

(c) $f'(x) = \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} \left(= \frac{1-2x^2}{\sqrt{1-x^2}}\right)$ *A1*

Note: This may be seen in part (b).

$f'(x) = 0 \Rightarrow 1 - 2x^2 = 0$ *M1*

$x = \pm\frac{1}{\sqrt{2}}$ *A1*

[3 marks]

(d) y-coordinates of the Max Min Points are $y = \pm\frac{1}{2}$ *M1A1*

so range of $f(x)$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$ *A1*

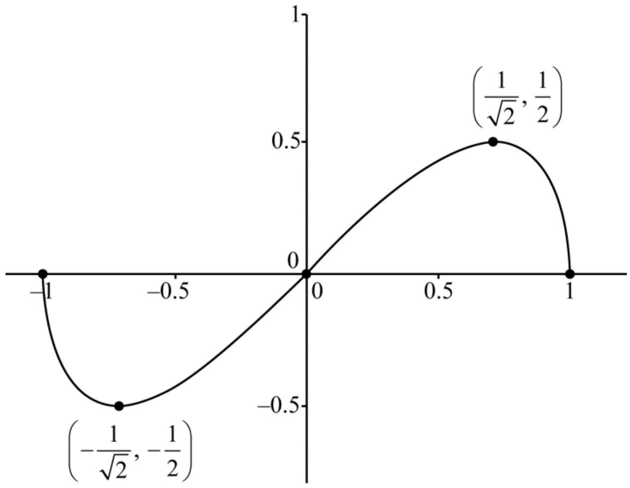
Note: Allow FT from (c) if values of x, within the domain, are used.

[3 marks]

continued…

Question 12 continued

(e)



Shape: The graph of an odd function, on the given domain, s-shaped, where the max(min) is the right(left) of $0.5(-0.5)$ ***A1***
x-intercepts ***A1***
turning points ***A1***

[3 marks]

(f) area $= \int_0^1 x\sqrt{1-x^2}\,\mathrm{d}x$ ***(M1)***

attempt at "backwards chain rule" or substitution ***M1***

$$= -\frac{1}{2}\int_0^1 (-2x)\sqrt{1-x^2}\,\mathrm{d}x$$

$$= \left[\frac{2}{3}\left(1-x^2\right)^{\frac{3}{2}} \cdot -\frac{1}{2}\right]_0^1$$ ***A1***

$$= \left[-\frac{1}{3}\left(1-x^2\right)^{\frac{3}{2}}\right]_0^1$$

$$= 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$$ ***A1***

[4 marks]

(g) $\int_{-1}^{1}\left|x\sqrt{1-x^2}\right|\mathrm{d}x > 0$ ***R1***

$\left|\int_{-1}^{1} x\sqrt{1-x^2}\,\mathrm{d}x\right| = 0$ ***R1***

so $\int_{-1}^{1}\left|x\sqrt{1-x^2}\right|\mathrm{d}x > \left|\int_{-1}^{1} x\sqrt{1-x^2}\,\mathrm{d}x\right| = 0$ ***AG***

[2 marks]

Total [20 marks]

13. (a) $\vec{BR} = \vec{BA} + \vec{AR} \;\left(= \vec{BA} + \frac{1}{2}\vec{AC}\right)$ ***(M1)***

$= (\boldsymbol{a} - \boldsymbol{b}) + \frac{1}{2}(\boldsymbol{c} - \boldsymbol{a})$

$= \frac{1}{2}\boldsymbol{a} - \boldsymbol{b} + \frac{1}{2}\boldsymbol{c}$ ***A1***

[2 marks]

(b) (i) $\mathrm{r}_{BR} = \boldsymbol{b} + \lambda\left(\frac{1}{2}\boldsymbol{a} - \boldsymbol{b} + \frac{1}{2}\boldsymbol{c}\right)\left(= \frac{\lambda}{2}\boldsymbol{a} + (1-\lambda)\,\boldsymbol{b} + \frac{\lambda}{2}\boldsymbol{c}\right)$ ***A1A1***

Note: Award ***A1A0*** if the $\mathrm{r} =$ is omitted in an otherwise correct expression/equation.

(ii) $\vec{AQ} = -\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} + \frac{1}{2}\boldsymbol{c}$ ***(A1)***

$\mathrm{r}_{AQ} = \boldsymbol{a} + \mu\left(-\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} + \frac{1}{2}\boldsymbol{c}\right)\left(= (1-\mu)\,\boldsymbol{a} + \frac{\mu}{2}\boldsymbol{b} + \frac{\mu}{2}\boldsymbol{c}\right)$ ***A1***

(iii) when $\vec{AQ}$ and $\vec{BP}$ intersect we will have $\mathrm{r}_{BR} = \mathrm{r}_{AQ}$ ***(M1)***

$\frac{\lambda}{2}\boldsymbol{a} + (1-\lambda)\,\boldsymbol{b} + \frac{\lambda}{2}\boldsymbol{c} = (1-\mu)\,\boldsymbol{a} + \frac{\mu}{2}\boldsymbol{b} + \frac{\mu}{2}\boldsymbol{c}$

attempt to equate the coefficients of the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ ***M1***

$$\left.\begin{aligned} \frac{\lambda}{2} &= 1-\mu \\ 1-\lambda &= \frac{\mu}{2} \\ \frac{\lambda}{2} &= \frac{\mu}{2} \end{aligned}\right\}$$ ***(A1)***

$\lambda = \frac{2}{3}$ or $\mu = \frac{2}{3}$ ***A1***

substituting parameters back into one of the equations ***M1***

$\vec{OG} = \frac{1}{2}\cdot\frac{2}{3}\boldsymbol{a} + \left(1 - \frac{2}{3}\right)\boldsymbol{b} + \frac{1}{2}\cdot\frac{2}{3}\boldsymbol{c} = \frac{1}{3}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$ ***AG***

[9 marks]

continued…

Question 13 continued

(c) $\vec{\mathrm{CP}} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} - \boldsymbol{c}$ ***(M1)A1***

so we have that $\mathrm{r}_{\mathrm{CP}} = \boldsymbol{c} + \beta\left(\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} - \boldsymbol{c}\right)$ and when $\beta = \frac{2}{3}$ the line passes through

the point G (*ie*, with position vector $\frac{1}{3}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$) ***R1***

hence $[\mathrm{AQ}], [\mathrm{BR}]$ and $[\mathrm{CP}]$ all intersect in G ***AG***

[3 marks]

continued…

Question 13 continued

(d) $\overrightarrow{OG} = \frac{1}{3}\left(\begin{pmatrix}1\\3\\1\end{pmatrix} + \begin{pmatrix}3\\7\\-5\end{pmatrix} + \begin{pmatrix}2\\2\\1\end{pmatrix}\right) = \begin{pmatrix}2\\4\\-1\end{pmatrix}$ ***A1***

Note:	This independent mark for the vector may be awarded wherever the vector is calculated.

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}2\\4\\-6\end{pmatrix} \times \begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}-6\\-6\\-6\end{pmatrix}$ ***M1A1***

$\overrightarrow{GX} = \alpha\begin{pmatrix}1\\1\\1\end{pmatrix}$ ***(M1)***

volume of Tetrahedron given by $\frac{1}{3} \times$ Area $\mathrm{ABC} \times \mathrm{GX}$

$= \frac{1}{3}\left(\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|\right) \times \mathrm{GX} = 12$ ***(M1)(A1)***

Note: Accept alternative methods, for example the use of a scalar triple product.

$= \frac{1}{6}\sqrt{(-6)^2 + (-6)^2 + (-6)^2} \times \sqrt{\alpha^2 + \alpha^2 + \alpha^2} = 12$ ***(A1)***

$= \frac{1}{6}6\sqrt{3}\,|\alpha|\sqrt{3} = 12$

$\Rightarrow |\alpha| = 4$ ***A1***

Note: Condone absence of absolute value.

this gives us the position of X as $\begin{pmatrix}2\\4\\-1\end{pmatrix} \pm \begin{pmatrix}4\\4\\4\end{pmatrix}$

$\mathrm{X}(6,\ 8,\ 3)$ or $(-2,\ 0,\ -5)$ ***A1***

Note: Award ***A1*** for either result.

[9 marks]

Total [23 marks]