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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

2 hours

Examination code

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the boxes provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

When the polynomial $3x^3 + ax + b$ is divided by $(x-2)$, the remainder is 2, and when divided by $(x+1)$, it is 5. Find the value of a and the value of b.

2. *[Maximum mark: 4]*

Four numbers are such that their mean is 13, their median is 14 and their mode is 15. Find the four numbers.

3. *[Maximum mark: 5]*

Consider $a = \log_2 3 \times \log_3 4 \times \log_4 5 \times \ldots \times \log_{31} 32$. Given that $a \in \mathbb{Z}$, find the value of a.

4. *[Maximum mark: 6]*

The equation $5x^3 + 48x^2 + 100x + 2 = a$ has roots r_1, r_2 and r_3.
Given that $r_1 + r_2 + r_3 + r_1 r_2 r_3 = 0$, find the value of a.

5. *[Maximum mark: 8]*

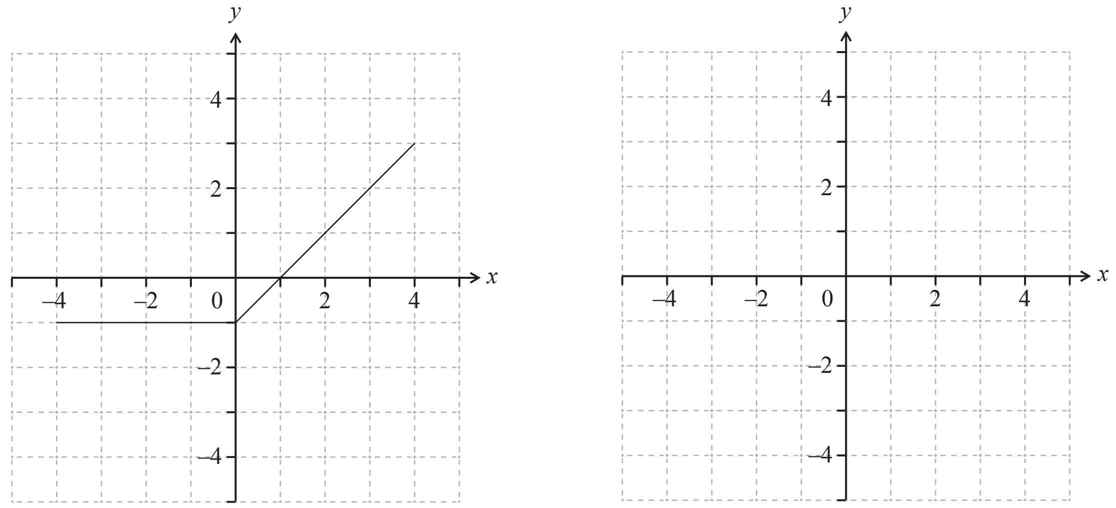
(a) Use the identity $\cos 2\theta = 2\cos^2\theta - 1$ to prove that $\cos\frac{1}{2}x = \sqrt{\frac{1+\cos x}{2}}$, $0 \le x \le \pi$. *[2]*

(b) Find a similar expression for $\sin\frac{1}{2}x$, $0 \le x \le \pi$. *[2]*

(c) Hence find the value of $\int_0^{\frac{\pi}{2}}\left(\sqrt{1+\cos x}+\sqrt{1-\cos x}\right)\mathrm{d}x$. *[4]*

6. *[Maximum mark: 6]*

The first set of axes below shows the graph of $y = f(x)$ for $-4 \le x \le 4$.



Let $g(x) = \int_{-4}^{x} f(t)\,\mathrm{d}t$ for $-4 \le x \le 4$.

(a) State the value of x at which $g(x)$ is a minimum. *[1]*

(b) On the second set of axes, sketch the graph of $y = g(x)$. *[5]*

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7. *[Maximum mark: 5]*

The triangle ABC is equilateral of side 3 cm. The point D lies on [BC] such that $\text{BD} = 1\text{cm}$.
Find $\cos \text{D}\hat{\text{A}}\text{C}$.

8. *[Maximum mark: 6]*

A body is moving in a straight line. When it is s metres from a fixed point O on the line its velocity, v, is given by $v = -\frac{1}{s^2}$, $s > 0$.

Find the acceleration of the body when it is 50 cm from O.

9. *[Maximum mark: 9]*

A curve has equation $\arctan x^2 + \arctan y^2 = \frac{\pi}{4}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. *[4]*

(b) Find the gradient of the curve at the point where $x = \frac{1}{\sqrt{2}}$ and $y < 0$. *[5]*

10. *[Maximum mark: 6]*

Given that $\sin x + \cos x = \frac{2}{3}$, find $\cos 4x$.

*Do **NOT** write solutions on this page.*

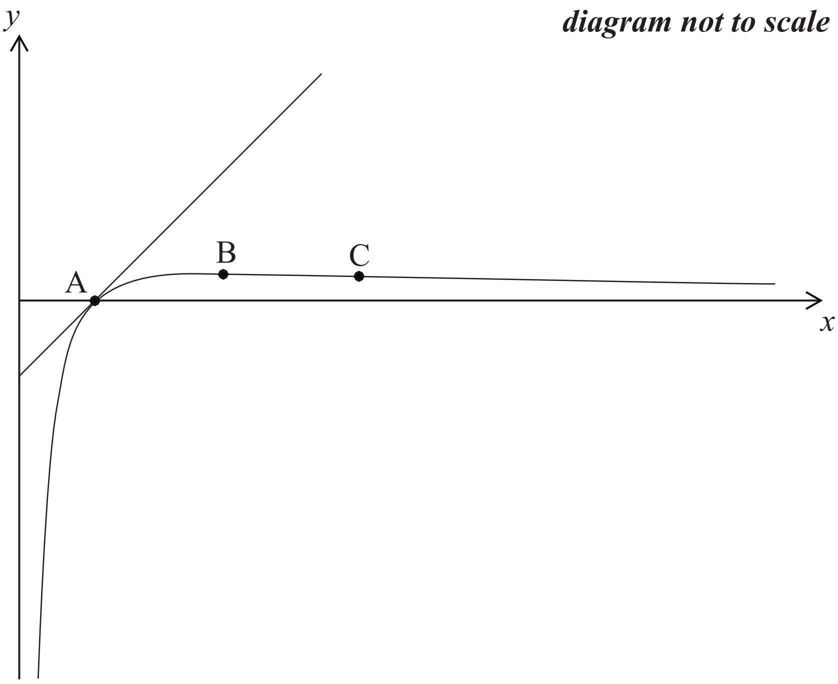
SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 21]*

Consider the function $f(x) = \dfrac{\ln x}{x}$, $x > 0$.

The sketch below shows the graph of $y = f(x)$ and its tangent at a point A.



(a) Show that $f'(x) = \dfrac{1 - \ln x}{x^2}$. *[2]*

(b) Find the coordinates of B, at which the curve reaches its maximum value. *[3]*

(c) Find the coordinates of C, the point of inflexion on the curve. *[5]*

The graph of $y = f(x)$ crosses the x-axis at the point A.

(d) Find the equation of the tangent to the graph of f at the point A. *[4]*

(e) Find the area enclosed by the curve $y = f(x)$, the tangent at A, and the line $x = \mathrm{e}$. *[7]*

12. *[Maximum mark: 22]*

(a) Show that the points $O(0, 0, 0)$, $A(6, 0, 0)$, $B\left(6, -\sqrt{24}, \sqrt{12}\right)$, $C\left(0, -\sqrt{24}, \sqrt{12}\right)$ form a square. *[3]*

(b) Find the coordinates of M, the mid-point of [OB]. *[1]*

(c) Show that an equation of the plane Π, containing the square OABC, is $y + \sqrt{2}z = 0$. *[3]*

(d) Find a vector equation of the line L, through M, perpendicular to the plane Π. *[3]*

(e) Find the coordinates of D, the point of intersection of the line L with the plane whose equation is $y = 0$. *[3]*

(f) Find the coordinates of E, the reflection of the point D in the plane Π. *[3]*

(g) (i) Find the angle $\text{O}\hat{\text{D}}\text{A}$.

(ii) State what this tells you about the solid OABCDE. *[6]*

13. *[Maximum mark: 17]*

A geometric sequence $\{u_n\}$, with complex terms, is defined by $u_{n+1} = (1+\mathrm{i})u_n$ and $u_1 = 3$.

(a) Find the fourth term of the sequence, giving your answer in the form $x + y\mathrm{i}$, $x, y \in \mathbb{R}$. *[3]*

(b) Find the sum of the first 20 terms of $\{u_n\}$, giving your answer in the form $a \times (1 + 2^m)$ where $a \in \mathbb{C}$ and $m \in \mathbb{Z}$ are to be determined. *[4]*

A second sequence $\{v_n\}$ is defined by $v_n = u_n u_{n+k}$, $k \in \mathbb{N}$.

(c) (i) Show that $\{v_n\}$ is a geometric sequence.

(ii) State the first term.

(iii) Show that the common ratio is independent of k. *[5]*

A third sequence $\{w_n\}$ is defined by $w_n = |u_n - u_{n+1}|$.

(d) (i) Show that $\{w_n\}$ is a geometric sequence.

(ii) State the geometrical significance of this result with reference to points on the complex plane. *[5]*

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