88127207

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 3 – DISCRETE MATHEMATICS

Thursday 8 November 2012 (morning)

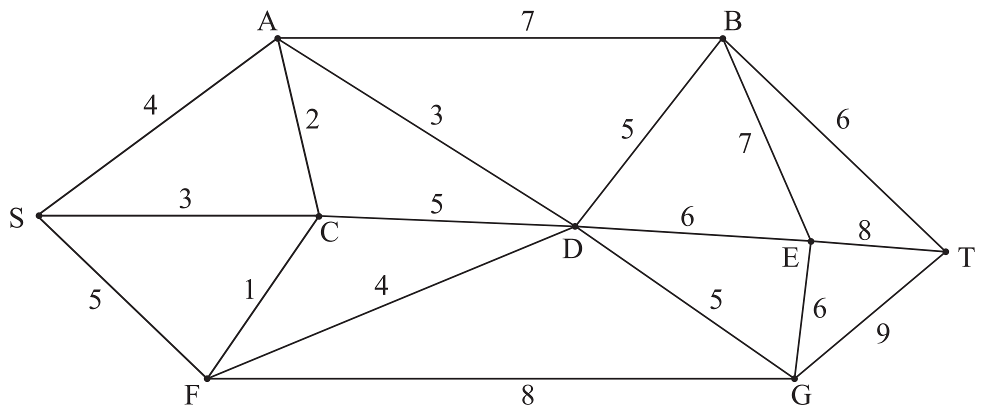
1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 14]*



In the graph given above, the numbers shown represent the distance along that edge.

(a) Using Dijkstra's algorithm, find the length of the shortest path from vertex S to vertex T. Write down this shortest path. *[6 marks]*

(b) (i) Does this graph have an Eulerian circuit? Justify your answer.

(ii) Does this graph have an Eulerian trail? Justify your answer. *[4 marks]*

(c) The graph above is now to be considered with the edges representing roads in a town and with the distances being the length of that road in kilometres. Huan is a postman and he has to travel along every road in the town to deliver letters to all the houses in that road. He has to start at the sorting office at S and also finish his route at S. Find the shortest total distance of such a route. Fully explain the reasoning behind your answer. *[4 marks]*

2. *[Maximum mark: 10]*

(a) Draw all the possible non-isomorphic simple graphs with three vertices. (No proofs are required.) *[4 marks]*

(b) Draw all the possible non-isomorphic trees with six vertices. (No proofs are required.) *[6 marks]*

3. *[Maximum mark: 18]*

Let the greatest common divisor of 861 and 957 be h.

(a) Using the Euclidean algorithm, find h. *[4 marks]*

(b) Hence find integers A and B such that $861A + 957B = h$. *[3 marks]*

(c) Using part (b), solve $287w \equiv 2 \pmod{319}$, where $w \in \mathbb{N}$, $w < 319$. *[5 marks]*

(d) Find the general solution to the diophantine equation $861x + 957y = 6$. *[6 marks]*

4. *[Maximum mark: 10]*

(a) State Fermat's little theorem. *[2 marks]*

(b) Let $x \in \mathbb{N}$. If $x \not\equiv 0 \pmod{5}$, $x \not\equiv 0 \pmod{13}$ and $x^{12} \equiv 4 \pmod{11}$, find the remainder when x^{12} is divided by 715. *[8 marks]*

5. *[Maximum mark: 8]*

G is a simple connected graph with four vertices and its complement $\overline{G}$ is also connected. Prove that G and $\overline{G}$ are isomorphic.