88127202

MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

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Wednesday 7 November 2012 (morning)

Examination code

8	8	1	2	–	7	2	0	2

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

Find the sum of all the multiples of 3 between 100 and 500.

2. *[Maximum mark: 4]*

Show that the quadratic equation $x^2 - (5-k)\,x - (k+2) = 0$ has two distinct real roots for all real values of k.

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3. *[Maximum mark: 5]*

Consider the matrix $\boldsymbol{A} = \begin{pmatrix} \ln x & \ln(5-x) \\ 2 & 3 \end{pmatrix}$, where $0 < x < 5$. Find the value of x for which $\boldsymbol{A}$ is singular.

4. *[Maximum mark: 6]*

A set of 15 observations has mean 11.5 and variance 9.3. One observation of 22.1 is considered unreliable and is removed. Find the mean and variance of the remaining 14 observations.

5. *[Maximum mark: 6]*

A metal rod 1 metre long is cut into 10 pieces, the lengths of which form a geometric sequence. The length of the longest piece is 8 times the length of the shortest piece. Find, to the nearest millimetre, the length of the shortest piece.

6. *[Maximum mark: 6]*

A particle moves along a straight line so that after t seconds its displacement s, in metres, satisfies the equation $s^2 + s - 2t = 0$. Find, in terms of s, expressions for its velocity and its acceleration.

7. *[Maximum mark: 8]*

Kathy plays a computer game in which she has to find the path through a maze within a certain time. The first time she attempts the game, the probability of success is known to be 0.75. In subsequent attempts, if Kathy is successful, the difficulty increases and the probability of success is half the probability of success on the previous attempt. However, if she is unsuccessful, the probability of success remains the same. Kathy plays the game three times consecutively.

(a) Find the probability that she is successful in all three games. *[2 marks]*

(b) Assuming that she is successful in the first game, find the probability that she is successful in exactly two games. *[6 marks]*

8. *[Maximum mark: 7]*

By using the substitution $x = \sin t$, find $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$.

9. *[Maximum mark: 7]*

Find the area of the region enclosed by the curves $y = x^3$ and $x = y^2 - 3$.

10. *[Maximum mark: 7]*

Let $\omega = \cos\theta + \mathrm{i}\sin\theta$. Find, in terms of θ, the modulus and argument of $(1-\omega^2)^*$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 18]*

The number of visitors that arrive at a museum every minute can be modelled by a Poisson distribution with mean 2.2.

(a) If the museum is open 6 hours daily, find the expected number of visitors in 1 day. *[2 marks]*

(b) Find the probability that the number of visitors arriving during an hour exceeds 100. *[3 marks]*

(c) Find the probability that the number of visitors in each of the 6 hours the museum is open exceeds 100. *[2 marks]*

The ages of the visitors to the museum can be modelled by a normal distribution with mean μ and variance σ^2. The records show that 29 % of the visitors are under 35 years of age and 23 % are at least 55 years of age.

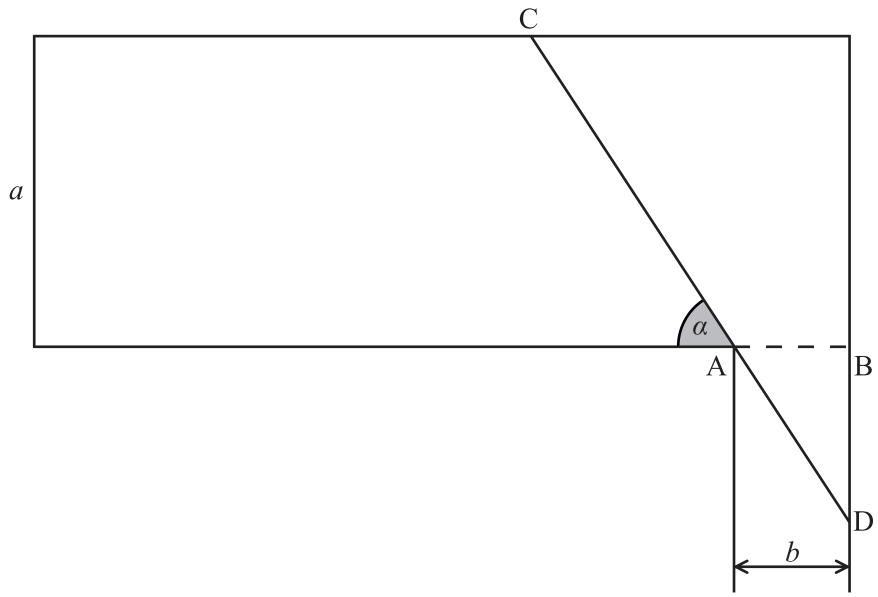
(d) Find the values of μ and σ. *[6 marks]*

(e) One day, 100 visitors under 35 years of age come to the museum. Estimate the number of visitors under 50 years of age that were at the museum on that day. *[5 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 18]*

The diagram shows the plan of an art gallery a metres wide. [AB] represents a doorway, leading to an exit corridor b metres wide. In order to remove a painting from the art gallery, CD (denoted by L) is measured for various values of α, as represented in the diagram.



(a) If α is the angle between [CD] and the wall, show that $L = \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha}$, $0 < \alpha < \frac{\pi}{2}$. *[3 marks]*

(b) If $a = 5$ and $b = 1$, find the maximum length of a painting that can be removed through this doorway. *[4 marks]*

Let $a = 3k$ and $b = k$.

(c) Find $\frac{dL}{d\alpha}$. *[3 marks]*

(d) Find, in terms of k, the maximum length of a painting that can be removed from the gallery through this doorway. *[6 marks]*

(e) Find the minimum value of k if a painting 8 metres long is to be removed through this doorway. *[2 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 24]*

Consider the planes $\pi_1 : x - 2y - 3z = 2$ and $\pi_2 : 2x - y - z = k$.

(a) Find the angle between the planes π_1 and π_2. *[4 marks]*

(b) The planes π_1 and π_2 intersect in the line L_1. Show that the vector equation of L_1 is $\boldsymbol{r} = \begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$. *[5 marks]*

(c) The line L_2 has Cartesian equation $5 - x = y + 3 = 2 - 2z$. The lines L_1 and L_2 intersect at a point X. Find the coordinates of X. *[5 marks]*

(d) Determine a Cartesian equation of the plane π_3 containing both lines L_1 and L_2. *[5 marks]*

(e) Let Y be a point on L_1 and Z be a point on L_2 such that XY is perpendicular to YZ and the area of the triangle XYZ is 3. Find the perimeter of the triangle XYZ. *[5 marks]*

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