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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Monday 7 May 2012 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 6]*

Use L'Hôpital's Rule to find $\lim_{x \to 0} \frac{e^x - 1 - x\cos x}{\sin^2 x}$.

2. *[Maximum mark: 21]*

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{1+x}$, where $x > -1$ and $y = 1$ when $x = 0$.

(a) Use Euler's method, with a step length of 0.1, to find an approximate value of y when $x = 0.5$. *[7 marks]*

(b) (i) Show that $\frac{d^2y}{dx^2} = \frac{2y^3 - y^2}{(1+x)^2}$.

(ii) Hence find the Maclaurin series for y, up to and including the term in x^2. *[8 marks]*

(c) (i) Solve the differential equation.

(ii) Find the value of a for which $y \to \infty$ as $x \to a$. *[6 marks]*

3. *[Maximum mark: 7]*

Find the general solution of the differential equation $t\frac{dy}{dt} = \cos t - 2y$, for $t > 0$.

4. *[Maximum mark: 15]*

The sequence $\{u_n\}$ is defined by $u_n = \dfrac{3n+2}{2n-1}$, for $n \in \mathbb{Z}^+$.

(a) Show that the sequence converges to a limit L, the value of which should be stated. *[3 marks]*

(b) Find the least value of the integer N such that $|u_n - L| < \varepsilon$, for all $n > N$ where

(i) $\varepsilon = 0.1$;

(ii) $\varepsilon = 0.00001$. *[4 marks]*

(c) For each of the sequences $\left\{\dfrac{u_n}{n}\right\}$, $\left\{\dfrac{1}{2u_n - 2}\right\}$ and $\left\{(-1)^n u_n\right\}$, determine whether or not it converges. *[6 marks]*

(d) Prove that the series $\sum_{n=1}^{\infty}(u_n - L)$ diverges. *[2 marks]*

5. *[Maximum mark: 11]*

(a) Find the set of values of k for which the improper integral $\int_2^{\infty} \dfrac{\mathrm{d}x}{x(\ln x)^k}$ converges. *[6 marks]*

(b) Show that the series $\sum_{r=2}^{\infty} \dfrac{(-1)^r}{r \ln r}$ is convergent but not absolutely convergent. *[5 marks]*