M12/5/MATHL/HP2/ENG/TZ2/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 4 May 2012 (morning)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	2	0	6

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.

(a) Find the first term and the common difference. *[4 marks]*

(b) Find the smallest value of n such that the sum of the first n terms is greater than 600. *[3 marks]*

2. *[Maximum mark: 5]*

The random variable X has the distribution $B(30, p)$. Given that $E(X) = 10$, find

(a) the value of p; *[1 mark]*

(b) $P(X = 10)$; *[2 marks]*

(c) $P(X \geq 15)$. *[2 marks]*

3. *[Maximum mark: 8]*

Consider a triangle ABC with $\text{B}\hat{\text{A}}\text{C} = 45.7^\circ$, $\text{AB} = 9.63 \text{ cm}$ and $\text{BC} = 7.5 \text{ cm}$.

(a) By drawing a diagram, show why there are two triangles consistent with this information. *[2 marks]*

(b) Find the possible values of AC. *[6 marks]*

4. *[Maximum mark: 6]*

Fifteen boys and ten girls sit in a single line.

(a) In how many ways can they be seated in a single line so that the boys and girls are in two separate groups? *[3 marks]*

(b) Two boys and three girls are selected to go the theatre. In how many ways can this selection be made? *[3 marks]*

5. *[Maximum mark: 5]*

The random variable X has the distribution $\text{Po}(m)$.
Given that $\text{P}(X = 5) = \text{P}(X = 3) + \text{P}(X = 4)$, find

(a) the value of m; *[3 marks]*

(b) $\text{P}(X > 2)$. *[2 marks]*

6. *[Maximum mark: 8]*

(a) Sketch the curve $y = \dfrac{\cos x}{\sqrt{x^2+1}}$, $-4 \le x \le 4$ showing clearly the coordinates of the x-intercepts, any maximum points and any minimum points. *[4 marks]*

(b) Write down the gradient of the curve at $x = 1$. *[1 mark]*

(c) Find the equation of the normal to the curve at $x = 1$. *[3 marks]*

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7. *[Maximum mark: 5]*

The probability density function of a continuous random variable X is given by

$$f(x) = \frac{1}{1+x^4}\,,\ 0\,''\ x''\ a\,.$$

(a) Find the value of a. *[3 marks]*

(b) Find the mean of X. *[2 marks]*

8. *[Maximum mark: 8]*

Each time a ball bounces, it reaches 95 % of the height reached on the previous bounce. Initially, it is dropped from a height of 4 metres.

(a) What height does the ball reach after its fourth bounce? *[2 marks]*

(b) How many times does the ball bounce before it no longer reaches a height of 1 metre? *[3 marks]*

(c) What is the total distance travelled by the ball? *[3 marks]*

9. *[Maximum mark: 8]*

Two discs, one of radius 8 cm and one of radius 5 cm, are placed such that they touch each other. A piece of string is wrapped around the discs. This is shown in the diagram below.

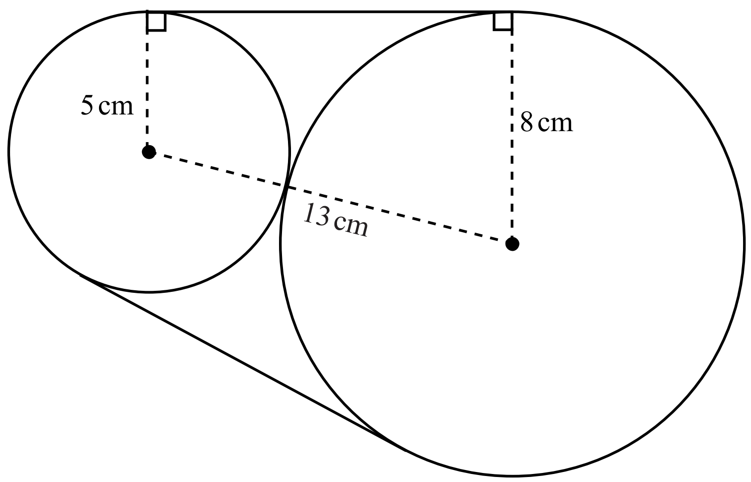

diagram not to scale

Calculate the length of string needed to go around the discs.

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SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 14]*

A market stall sells apples, pears and plums.

(a) The weights of the apples are normally distributed with a mean of 200 grams and a standard deviation of 25 grams.

(i) Given that there are 450 apples on the stall, what is the expected number of apples with a weight of more than 225 grams?

(ii) Given that 70 % of the apples weigh less than m grams, find the value of m. *[5 marks]*

(b) The weights of the pears are normally distributed with a mean of $\propto$ grams and a standard deviation of σ grams. Given that 8 % of these pears have a weight of more than 270 grams and 15 % have a weight less than 250 grams, find $\propto$ and σ. *[6 marks]*

(c) The weights of the plums are normally distributed with a mean of 80 grams and a standard deviation of 4 grams. 5 plums are chosen at random. What is the probability that exactly 3 of them weigh more than 82 grams? *[3 marks]*

*Do **NOT** write solutions on this page.*

11. *[Maximum mark: 24]*

(a) Find the values of k for which the following system of equations has no solutions and the value of k for the system to have an infinite number of solutions.

$$x - 3y + z = 3$$
$$x + 5y - 2z = 1$$
$$16y - 6z = k$$

[5 marks]

(b) Given that the system of equations can be solved, find the solutions in the form of a vector equation of a line, $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$, where the components of $\boldsymbol{b}$ are integers. *[7 marks]*

(c) The plane ÷ is parallel to both the line in part (b) and the line $\frac{x-4}{3} = \frac{y-6}{-2} = \frac{z-2}{0}$.

Given that ÷ contains the point $(1, 2, 0)$, show that the Cartesian equation of ÷ is $16x + 24y - 11z = 64$. *[5 marks]*

(d) The z-axis meets the plane ÷ at the point P. Find the coordinates of P. *[2 marks]*

(e) Find the angle between the line $\frac{x-2}{3} = \frac{y+5}{4} = \frac{z}{2}$ and the plane ÷ . *[5 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 22]*

A particle moves in a straight line with velocity v metres per second. At any time t seconds, $0 \le t < \frac{3\pi}{4}$, the velocity is given by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} + v^2 + 1 = 0$. It is also given that $v = 1$ when $t = 0$.

(a) Find an expression for v in terms of t. *[7 marks]*

(b) Sketch the graph of v against t, clearly showing the coordinates of any intercepts, and the equations of any asymptotes. *[3 marks]*

(c) (i) Write down the time T at which the velocity is zero.

(ii) Find the distance travelled in the interval $[0, T]$. *[3 marks]*

(d) Find an expression for s, the displacement, in terms of t, given that $s = 0$ when $t = 0$. *[5 marks]*

(e) Hence, or otherwise, show that $s = \frac{1}{2}\ln\frac{2}{1+v^2}$. *[4 marks]*

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