M12/5/MATHL/HP2/ENG/TZ1/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 4 May 2012 (morning)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	2	0	4

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Given that the graph of $y = x^3 - 6x^2 + kx - 4$ has exactly one point at which the gradient is zero, find the value of k.

2. *[Maximum mark: 4]*

The probability density function of the continuous random variable X is given by

$$f(x)=\begin{cases} k2^{\frac{1}{x}}, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Find the expected value of X.

3. *[Maximum mark: 7]*

A team of 6 players is to be selected from 10 volleyball players, of whom 8 are boys and 2 are girls.

(a) In how many ways can the team be selected? *[2 marks]*

(b) In how many of these selections is exactly one girl in the team? *[3 marks]*

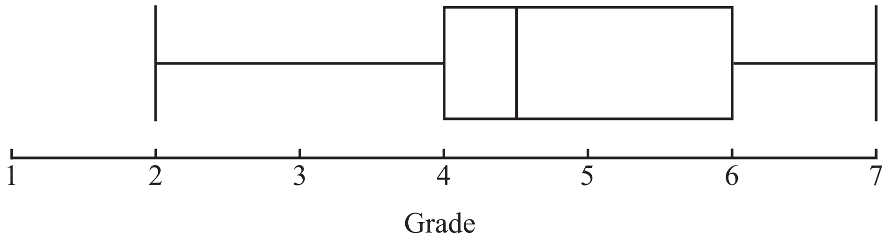
(c) If the selection of the team is made at random, find the probability that exactly one girl is in the team. *[2 marks]*

4. *[Maximum mark: 5]*

The planes $2x+3y-z=5$ and $x-y+2z=k$ intersect in the line $5x+1=9-5y=-5z$.
Find the value of k.

5. *[Maximum mark: 5]*

The box and whisker plot below illustrates the IB grades obtained by 100 students.



IB grades can only take integer values.

(a) How many students obtained a grade of more than 4? *[1 mark]*

(b) State, with reasons, the maximum possible number and minimum possible number of students who obtained a 4 in the exam. *[4 marks]*

6. *[Maximum mark: 5]*

Let $f(x) = \ln x$. The graph of f is transformed into the graph of the function g by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the x-axis. Find an expression for $g(x)$, giving your answer as a single logarithm.

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7. *[Maximum mark: 5]*

A fisherman notices that in any hour of fishing, he is equally likely to catch exactly two fish, as he is to catch less than two fish. Assuming the number of fish caught can be modelled by a Poisson distribution, calculate the expected value of the number of fish caught when he spends four hours fishing.

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8. *[Maximum mark: 9]*

A cone has height h and base radius r. Deduce the formula for the volume of this cone by rotating the triangular region, enclosed by the line $y = h - \frac{h}{r}x$ and the coordinate axes, through 2π about the y-axis.

9. *[Maximum mark: 7]*

Find the constant term in the expansion of $\left(x-\frac{2}{x}\right)^4\left(x^2+\frac{2}{x}\right)^3$.

10. *[Maximum mark: 8]*

A triangle is formed by the three lines $y = 10 - 2x$, $y = mx$ and $y = -\frac{1}{m}x$, where $m > \frac{1}{2}$.

Find the value of m for which the area of the triangle is a minimum.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 14]*

The function $f(x) = 3\sin x + 4\cos x$ is defined for $0 < x < 2\pi$.

(a) Write down the coordinates of the minimum point on the graph of f. *[1 mark]*

(b) The points $\mathrm{P}(p, 3)$ and $\mathrm{Q}(q, 3)$, $q > p$, lie on the graph of $y = f(x)$. Find p and q. *[2 marks]*

(c) Find the coordinates of the point, on $y = f(x)$, where the gradient of the graph is 3. *[4 marks]*

(d) Find the coordinates of the point of intersection of the normals to the graph at the points P and Q. *[7 marks]*

12. *[Maximum mark: 22]*

A ski resort finds that the mean number of accidents on any given weekday (Monday to Friday) is 2.2 . The number of accidents can be modelled by a Poisson distribution.

(a) Find the probability that in a certain week (Monday to Friday only)

(i) there are fewer than 12 accidents;

(ii) there are more than 8 accidents, given that there are fewer than 12 accidents. *[6 marks]*

Due to the increased usage, it is found that the probability of more than 3 accidents in a day at the weekend (Saturday and Sunday) is 0.24.

(b) Assuming a Poisson model,

(i) calculate the mean number of accidents per day at the weekend (Saturday and Sunday);

(ii) calculate the probability that, in the four weekends in February, there will be more than 5 accidents during at least two of the weekends. *[10 marks]*

It is found that 20 % of skiers having accidents are at least 25 years of age and 40 % are under 18 years of age.

(c) Assuming that the ages of skiers having accidents are normally distributed, find the mean age of skiers having accidents. *[6 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 24]*

The coordinates of points A, B and C are given as $(5, -2, 5)$, $(5, 4, -1)$ and $(-1, -2, -1)$ respectively.

(a) Show that $\text{AB} = \text{AC}$ and that $\text{B}\hat{\text{A}}\text{C} = 60^\circ$. *[4 marks]*

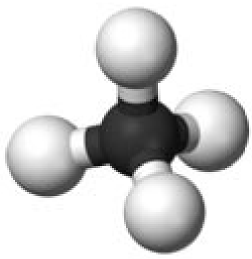
(b) Find the Cartesian equation of Π, the plane passing through A, B, and C. *[4 marks]*

(c) (i) Find the Cartesian equation of Π_1, the plane perpendicular to (AB) passing through the midpoint of [AB] .

(ii) Find the Cartesian equation of Π_2, the plane perpendicular to (AC) passing through the midpoint of [AC]. *[4 marks]*

(d) Find the vector equation of L, the line of intersection of Π_1 and Π_2, and show that it is perpendicular to Π. *[3 marks]*

A methane molecule consists of a carbon atom with four hydrogen atoms symmetrically placed around it in three dimensions.



The positions of the centres of three of the hydrogen atoms are A, B and C as given. The position of the centre of the fourth hydrogen atom is D.

(e) Using the fact that $\text{AB} = \text{AD}$, show that the coordinates of one of the possible positions of the fourth hydrogen atom is $(-1, 4, 5)$. *[3 marks]*

(f) Letting D be $(-1, 4, 5)$, show that the coordinates of G, the position of the centre of the carbon atom, are $(2, 1, 2)$. Hence calculate $\text{D}\hat{\text{G}}\text{A}$, the bonding angle of carbon. *[6 marks]*

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