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MARKSCHEME

November 2011

MATHEMATICS

Higher Level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

The method of dealing with accuracy errors on a whole paper basis by means of the Accuracy Penalty ***(AP)*** *no longer applies.*

Instructions to examiners about such numerical issues will be provided on a question by question basis within the framework of mathematical correctness, numerical understanding and contextual appropriateness.

The rubric on the front page of each question paper is given for the guidance of candidates. The markscheme (MS) may contain instructions to examiners in the form of "Accept answers which round to n significant figures (sf)". Where candidates state answers, required by the question, to fewer than n sf, award ***A0****. Some intermediate numerical answers may be required by the MS but not by the question. In these cases only award the mark(s) if the candidate states the answer exactly or to at least 2sf.*

11 Crossed out work

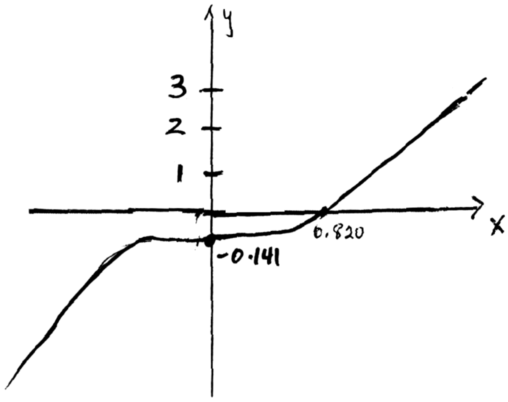
If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

SECTION A

1. (a)



A1A1A1

> **Note:** Award ***A1*** for shape,
> ***A1*** for x-intercept is 0.820, accept sin(–3) or –sin(3)
> ***A1*** for y-intercept is -0.141.

(b) $A = \int_0^{0.8202} \left| x + \sin(x-3) \right| \mathrm{d}x \approx 0.0816$ sq units *(M1)A1*

[5 marks]

2. $\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix}$ *(M1)*

$= a\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix}$

$= a(a^2-1)-(a-1)+(1-a)$ *(A1)*

$= a^3 - 3a + 2$ ***A1***

set $a^3 - 3a + 2 = 0$ ***M1***

$\Rightarrow a = -2$; $a = 1$ ***A1A1***

hence the system has a unique solution for all reals such that

$a \neq -2$; $a \neq 1$ ***R1***

> **Note:** Award ***R1*** for their values of a.

[7 marks]

3. (a) $m = \frac{300}{60} = 5$ *(A1)*

$\mathrm{P}(X = 0) = 0.00674$ ***A1***

or e^{-5}

(b) $\mathrm{E}(X) = 5 \times 2 = 10$ ***A1***

(c) $\mathrm{P}(X > 10) = 1 - \mathrm{P}(X \leq 10)$ *(M1)*

$= 0.417$ ***A1***

[5 marks]

4. (a) $\tan\left(\arctan\frac{1}{2}-\arctan\frac{1}{3}\right)=\tan(\arctan a)$ ***(M1)***

$a=0.14285\ldots=\frac{1}{7}$ ***(A1)A1***

(b) $\arctan\left(\frac{1}{7}\right)=\arcsin(x)\Rightarrow x=\sin\left(\arctan\frac{1}{7}\right)\approx 0.141$ ***(M1)A1***

Note: Accept exact value of $\left(\frac{1}{\sqrt{50}}\right)$.

[5 marks]

5. (a) $X \sim \mathrm{B}(5, 0.1)$ ***(M1)***

$\mathrm{P}(X=2)=0.0729$ ***A1***

(b) $\mathrm{P}(X\geq 1)=1-\mathrm{P}(X=0)$ ***(M1)***

$0.9<1-\left(\frac{9}{10}\right)^n$ ***(M1)***

$n>\frac{\ln 0.1}{\ln 0.9}$

$n=22$ days ***A1***

[5 marks]

6. **METHOD 1**

$\arg(z_1 z_2) = \frac{5\pi}{6}$ (150°) ***(A1)***

$\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ (90°) ***(A1)***

$\Rightarrow \arg(z_1) + \arg(z_2) = \frac{5\pi}{6}$; $\arg(z_1) - \arg(z_2) = \frac{\pi}{2}$ ***M1***

solving simultaneously

$\arg(z_1) = \frac{2\pi}{3}$ (120°) and $\arg(z_2) = \frac{\pi}{6}$ (30°) ***A1A1***

Note: Accept decimal approximations of the radian measures.

$|z_1 z_2| = 2 \Rightarrow |z_1||z_2| = 2$; $\left|\frac{z_1}{z_2}\right| = 2 \Rightarrow \frac{|z_1|}{|z_2|} = 2$ ***M1***

solving simultaneously

$|z_1| = 2$; $|z_2| = 1$ ***A1***

[7 marks]

METHOD 2

$z_1 = 2iz_2$ $\quad 2iz_2{}^2 = -\sqrt{3} + i$ ***(M1)***

$z_2{}^2 = \frac{-\sqrt{3}+i}{2i}$ ***A1***

$z_2 = \sqrt{\frac{-\sqrt{3}+i}{2i}} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ or $e^{\frac{\pi}{6}i}$ ***(M1)(A1)***

(allow $0.866 + 0.5i$ or $e^{0.524i}$)

$z_1 = -1 + \sqrt{3}i$ or $2e^{\frac{2\pi}{3}i}$ – (allow $-1+1.73i$ or $2e^{2.09i}$) ***(A1)***

z_1 modulus = 2, argument = $\frac{2\pi}{3}$ ***A1***

z_2 modulus = 1, argument = $\frac{\pi}{6}$ ***A1***

Note: Accept degrees and decimal approximations to radian measure.

[7 marks]

7. (a) for the series to have a finite sum, $\left|\frac{2x}{x+1}\right| < 1$ ***R1***

(sketch from gdc or algebraic method) ***M1***

S_∞ exists when $-\frac{1}{3} < x < 1$ ***A1A1***

Note: Award ***A1*** for bounds and ***A1*** for strict inequalities.

(b) $S_\infty = \dfrac{\frac{2x}{x+1}}{1-\frac{2x}{x+1}} = \dfrac{2x}{1-x}$ ***M1A1***

[6 marks]

8. (a) $y = \dfrac{1}{1+e^{-x}}$

$y(1+e^{-x}) = 1$ ***M1***

$1+e^{-x} = \frac{1}{y} \Rightarrow e^{-x} = \frac{1}{y} - 1$ ***A1***

$\Rightarrow x = -\ln\left(\frac{1}{y} - 1\right)$ ***A1***

$f^{-1}(x) = -\ln\left(\frac{1}{x} - 1\right) \quad \left(= \ln\left(\frac{x}{1-x}\right)\right)$ ***A1***

domain: $0 < x < 1$ ***A1A1***

Note: Award ***A1*** for endpoints and ***A1*** for strict inequalities.

(b) 0.659 ***A1***

[7 marks]

9. $V = \frac{\pi}{3} r^2 h$

$\frac{dV}{dt} = \frac{\pi}{3}\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$ ***M1A1A1***

at the given instant

$\frac{dV}{dt} = \frac{\pi}{3}\left[2(40)(200)\left(-\frac{1}{2}\right) + 40^2(3)\right]$ ***M1***

$= \frac{-3200\pi}{3} = -3351.03\ldots \approx -3350$ ***A1***

hence, the volume is decreasing (at approximately 3350 mm^3 per century) ***R1***

[6 marks]

10. **METHOD 1**

$$\frac{2-\mathrm{i}}{1+\mathrm{i}}=\frac{1-3\mathrm{i}}{2}$$ ***A1***

$$\frac{6+8\mathrm{i}}{u+\mathrm{i}}\times\frac{u-\mathrm{i}}{u-\mathrm{i}}=\frac{6u+8+(8u-6)\mathrm{i}}{u^2+1}$$ ***M1A1***

$$\Rightarrow\frac{2-\mathrm{i}}{1+\mathrm{i}}-\frac{6+8u}{u+\mathrm{i}}=\frac{1}{2}-\frac{6u+8}{u^2+1}-\left(\frac{3}{2}+\frac{8u-6}{u^2+1}\right)\mathrm{i}$$

$\operatorname{Im} z=\operatorname{Re} z$

$$\Rightarrow\frac{1}{2}-\frac{6u+8}{u^2+1}=-\frac{3}{2}-\frac{8u-6}{u^2+1}$$ ***A1***

(sketch from gdc, or algebraic method) ***(M1)***

$u=-3$; $u=2$ ***A1A1*** ***N2***

[7 marks]

METHOD 2

$$\frac{2-\mathrm{i}}{1+\mathrm{i}}-\frac{6+8\mathrm{i}}{u+\mathrm{i}}=\frac{(2-\mathrm{i})(u+\mathrm{i})-(1+\mathrm{i})(6+8\mathrm{i})}{(u-1)+\mathrm{i}(u+1)}$$ ***M1A1***

$$=\frac{(2-\mathrm{i})(u+\mathrm{i})-(1+\mathrm{i})(6+8\mathrm{i})}{(u-1)+\mathrm{i}(u+1)}\cdot\frac{(u-1)-\mathrm{i}(u+1)}{(u-1)-\mathrm{i}(u+1)}$$ ***M1***

$$=\frac{u^2-12u-15+\mathrm{i}(-3u^2-16u+9)}{2(u^2+1)}$$ ***A1***

$\operatorname{Re} z=\operatorname{Im} z\Rightarrow u^2-12u-15=-3u^2-16u+9$ ***M1***

$u=-3$; $u=2$ ***A1A1*** ***N2***

[7 marks]

SECTION B

11. (a) $X \sim \mathrm{N}(60.33, 1.95^2)$

$\mathrm{P}(X < x) = 0.2 \Rightarrow x = 58.69$ m ***(M1)A1***

[2 marks]

(b) $z = -0.8416\ldots$ ***(A1)***

$-0.8416 = \dfrac{56.52 - 59.39}{\sigma}$ ***(M1)***

$\sigma \approx 3.41$ ***A1***

[3 marks]

(c) Jan $X \sim \mathrm{N}(60.33, 1.95^2)$; Sia $X \sim \mathrm{N}(59.50, 3.00^2)$

(i) Jan: $\mathrm{P}(X > 65) \approx 0.00831$ ***(M1)A1***

Sia: $\mathrm{P}(Y > 65) \approx 0.0334$ ***A1***

Sia is more likely to qualify ***R1***

Note: Only award ***R1*** if ***(M1)*** has been awarded.

(ii) Jan: $\mathrm{P}(X \ge 1) = 1 - \mathrm{P}(X = 0)$ ***(M1)***

$= 1 - (1 - 0.00831\ldots)^3 \approx 0.0247$ ***(M1)A1***

Sia: $\mathrm{P}(Y \ge 1) = 1 - \mathrm{P}(Y = 0) = 1 - (1 - 0.0334\ldots)^3 \approx 0.0968$ ***A1***

Note: Accept 0.0240 and 0.0969.

hence, $\mathrm{P}(X \ge 1 \text{ and } Y \ge 1) = 0.0247 \times 0.0968 = 0.00239$ ***(M1)A1***

[10 marks]

Total [15 marks]

12. (a) $S_{2n}=\frac{2n}{2}\left(2(8)+(2n-1)\frac{1}{4}\right)$ ***(M1)***

$=n\left(16+\frac{2n-1}{4}\right)$ ***A1***

$S_{3n}=\frac{3n}{2}\left(2\times 8+(3n-1)\frac{1}{4}\right)$ ***(M1)***

$=\frac{3n}{2}\left(16+\frac{3n-1}{4}\right)$ ***A1***

$S_{2n}=S_{3n}-S_{2n}\Rightarrow 2S_{2n}=S_{3n}$ ***M1***

solve $2S_{2n}=S_{3n}$

$\Rightarrow 2n\left(16+\frac{2n-1}{4}\right)=\frac{3n}{2}\left(16+\frac{3n-1}{4}\right)$ ***A1***

$\left(\Rightarrow 2\left(16+\frac{2n-1}{4}\right)=\frac{3}{2}\left(16+\frac{3n-1}{4}\right)\right)$

(gdc or algebraic solution) ***(M1)***

$n=63$ ***A2***

[9 marks]

(b) $(a_1-a_2)^2+(a_2-a_3)^2+(a_3-a_4)^2+\ldots$

$=(a_1-a_1r)^2+(a_1r-a_1r^2)^2+(a_1r^2-a_1r^3)+\ldots$ ***M1A1***

$=\left[a_1(1-r)\right]^2+\left[a_1r(1-r)\right]^2+\left[a_1r^2(1-r)\right]^2+\ldots+\left[a_1r^{n-1}(1-r)\right]^2$ ***(A1)***

Note: This ***A1*** is for the expression for the last term.

$=a_1{}^2(1-r)^2+a_1{}^2r^2(1-r)^2+a_1{}^2r^4(1-r)^2+\ldots+a_1{}^2r^{2n-2}(1-r)^2$ ***A1***

$=a_1{}^2(1-r)^2(1+r^2+r^4+\ldots+r^{2n-2})$ ***A1***

$=a_1{}^2(1-r)^2\left(\frac{1-r^{2n}}{1-r^2}\right)$ ***M1A1***

$=\frac{a_1{}^2(1-r)(1-r^{2n})}{1+r}$ ***AG***

[7 marks]

Total [16 marks]

13. (a) **METHOD 1**

solving simultaneously (gdc) ***(M1)***

$x = 1 + 2z$; $y = -1 - 5z$ ***A1A1***

$$L : \boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$$ ***A1A1A1***

Note: 1$^{\text{st}}$ ***A1*** is for $\boldsymbol{r} =$.

[6 marks]

METHOD 2

direction of line $= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix}$ (last two rows swapped) ***M1***

$= 2\boldsymbol{i} - 5\boldsymbol{j} + \boldsymbol{k}$ ***A1***

putting $z = 0$, a point on the line satisfies $2x + y = 1$, $3x + y = 2$ ***M1***

i.e. $(1, -1, 0)$ ***A1***

the equation of the line is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$$ ***A1A1***

Note: Award ***A0A1*** if $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is missing.

[6 marks]

(b) $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$ ***M1***

$= 6\boldsymbol{i} - 12\boldsymbol{k}$ ***A1***

hence, $\boldsymbol{n} = \boldsymbol{i} - 2\boldsymbol{k}$

$\boldsymbol{n} \bullet \boldsymbol{a} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 1$ ***M1A1***

therefore $\boldsymbol{r} \bullet \boldsymbol{n} = \boldsymbol{a} \bullet \boldsymbol{n} \Rightarrow x - 2z = 1$ ***AG***

[4 marks]

continued …

Question 13 continued

(c) **METHOD 1**

$P = (-2, 4, 1)$, $Q = (x, y, z)$

$\overrightarrow{PQ} = \begin{pmatrix} x+2 \\ y-4 \\ z-1 \end{pmatrix}$ ***A1***

$\overrightarrow{PQ}$ is perpendicular to $3x + y - z = 2$

$\Rightarrow \overrightarrow{PQ}$ is parallel to $3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ ***R1***

$\Rightarrow x + 2 = 3t$; $y - 4 = t$; $z - 1 = -t$ ***A1***

$1 - z = t \Rightarrow x + 2 = 3 - 3z \Rightarrow x + 3z = 1$ ***A1***

solving simultaneously $x + 3z = 1$; $x - 2z = 1$ ***M1***

$5z = 0 \Rightarrow z = 0$; $x = 1$, $y = 5$ ***A1***

hence, $Q = (1, 5, 0)$

[6 marks]

METHOD 2

Line passing through PQ has equation

$$\mathbf{r} = \begin{matrix} -2 \\ 4 \\ 1 \end{matrix} + t \begin{matrix} 3 \\ 1 \\ -1 \end{matrix}$$ ***M1A1***

Meets π_3 when:

$-2 + 3t - 2(1 - t) = 1$ ***M1A1***

$t = 1$ ***A1***

Q has coordinates (1, 5, 0) ***A1***

[6 marks]

Total [16 marks]

14. (a) $\left|e^{i\theta}\right| \ \left(=|\cos\theta + i\sin\theta|\right) = \sqrt{\cos^2\theta + \sin^2\theta} = 1$ ***M1AG***

[1 mark]

(b) $z = \frac{1}{3}e^{i\theta}$ ***A1***

$|z| = \left|\frac{1}{3}e^{i\theta}\right| = \frac{1}{3}$ ***A1AG***

[2 marks]

(c) $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}e^{i\theta}}$ ***(M1)A1***

[2 marks]

(d) **EITHER**

$$S_\infty = \frac{1}{1-\frac{1}{3}\cos\theta - \frac{1}{3}i\sin\theta}$$ ***A1***

$$= \frac{1-\frac{1}{3}\cos\theta + \frac{1}{3}i\sin\theta}{\left(1-\frac{1}{3}\cos\theta - \frac{1}{3}i\sin\theta\right)\left(1-\frac{1}{3}\cos\theta + \frac{1}{3}i\sin\theta\right)}$$ ***M1A1***

$$= \frac{1-\frac{1}{3}\cos\theta + \frac{1}{3}i\sin\theta}{\left(1-\frac{1}{3}\cos\theta\right)^2 + \frac{1}{9}\sin^2\theta}$$ ***A1***

$$= \frac{1-\frac{1}{3}\cos\theta + \frac{1}{3}i\sin\theta}{1-\frac{2}{3}\cos\theta + \frac{1}{9}}$$ ***A1***

continued …

Question 14 continued

OR

$$S_\infty = \frac{1}{1-\frac{1}{3}\mathrm{e}^{\mathrm{i}\theta}}$$

$$= \frac{1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}}{\left(1-\frac{1}{3}\mathrm{e}^{\mathrm{i}\theta}\right)\left(1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}\right)}$$ ***M1A1***

$$= \frac{1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}}{1-\frac{1}{3}(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta})+\frac{1}{9}}$$ ***A1***

$$= \frac{1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}}{\frac{10}{9}-\frac{2}{3}\cos\theta}$$ ***A1***

$$= \frac{1-\frac{1}{3}(\cos\theta-\mathrm{i}\sin\theta)}{\frac{10}{9}-\frac{2}{3}\cos\theta}$$ ***A1***

THEN

taking imaginary parts on both sides

$$\frac{1}{3}\sin\theta+\frac{1}{9}\sin 2\theta+\ldots = \frac{\frac{1}{3}\sin\theta}{\frac{10}{9}-\frac{2}{3}\cos\theta}$$ ***M1A1A1***

$$= \frac{\sin\theta}{\frac{10}{9}-\frac{2}{3}\cos\theta}$$

$$\Rightarrow \sin\theta+\frac{1}{3}\sin 2\theta+\ldots = \frac{9\sin\theta}{10-6\cos\theta}$$ ***AG***

[8 marks]

Total ***[13 marks]***