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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Monday 9 May 2011 (morning)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

4 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 11]*

The weights of the oranges produced by a farm may be assumed to be normally distributed with mean 205 grams and standard deviation 10 grams.

(a) Find the probability that a randomly chosen orange weighs more than 200 grams. *[2 marks]*

(b) Five of these oranges are selected at random to be put into a bag. Find the probability that the combined weight of the five oranges is less than 1 kilogram. *[4 marks]*

(c) The farm also produces lemons whose weights may be assumed to be normally distributed with mean 75 grams and standard deviation 3 grams. Find the probability that the weight of a randomly chosen orange is more than three times the weight of a randomly chosen lemon. *[5 marks]*

2. *[Maximum mark: 10]*

The discrete random variable X has the distribution $\text{Geo}(p)$. The value of p is believed to be 0.3. In order to test this belief a random sample of 100 values of X was obtained, with the following results.

Value of X	1	2	3	4	5	≥ 6
Frequency	35	18	16	11	10	10

(a) State suitable hypotheses to test this belief. *[1 mark]*

(b) Carry out a χ^2-test at the 5 % significance level and, giving a reason, state your conclusion. *[9 marks]*

3. *[Maximum mark: 17]*

Ten friends try a diet which is claimed to reduce weight. They each weigh themselves before starting the diet, and after a month on the diet, with the following results.

Friend	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Weight before (kg)	68.4	70.9	74.7	65.4	59.4	69.0	73.9	62.6	68.3	58.2
Weight after (kg)	66.2	67.4	70.4	65.9	55.2	69.2	71.4	59.9	68.2	58.9

(a) Determine unbiased estimates of the mean and variance of the loss in weight achieved over the month by people using this diet. *[5 marks]*

(b) (i) State suitable hypotheses for testing whether or not this diet causes a mean loss in weight.

(ii) Determine the value of a suitable statistic for testing your hypotheses.

(iii) Find the 1 % critical value for your statistic and state your conclusion. *[6 marks]*

(c) One of the friends calculates a confidence interval for the mean loss in weight obtained by people using this diet for a month and he obtains $[0.26, 3.36]$. Find the confidence level of this interval. *[6 marks]*

4. *[Maximum mark: 10]*

The random variable X has a Poisson distribution with unknown mean μ. It is required to test the hypotheses

$$H_0 : \mu = 3 \text{ against } H_1 : \mu \neq 3 .$$

Let S denote the sum of 10 randomly chosen values of X. The critical region is defined as $(S \leq 22) \cup (S \geq 38)$.

(a) Calculate the significance level of the test. *[5 marks]*

(b) Given that the value of μ is actually 2.5, determine the probability of a Type II error. *[5 marks]*

5. *[Maximum mark: 12]*

(a) The random variable X has the negative binomial distribution $\text{NB}(3,\ p)$. Let $f(x)$ denote the probability that X takes the value x.

(i) Write down an expression for $f(x)$, and show that

$$\ln f(x) = 3\ln\left(\frac{p}{1-p}\right) + \ln(x-1) + \ln(x-2) + x\ln(1-p) - \ln 2\,.$$

(ii) State the domain of f.

(iii) The domain of f is extended to $]2,\ \infty[$. Show that

$$\frac{f'(x)}{f(x)} = \frac{1}{x-1} + \frac{1}{x-2} + \ln(1-p)\,.$$

[7 marks]

(b) Jo has a biased coin which has a probability of 0.35 of showing heads when tossed. She tosses this coin successively and the 3^{rd} head occurs on the Y^{th} toss. Use the result in part (a)(iii) to find the most likely value of Y. *[5 marks]*