N10/5/MATHL/HP3/ENG/TZ0/SP

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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Monday 15 November 2010 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 14]*

(a) A hospital specializes in treating overweight patients. These patients have weights that are independently, normally distributed with mean 200 kg and standard deviation 15 kg. The elevator in the hospital will break if the total weight of people inside it exceeds 1150 kg. Six patients enter the elevator. Find the probability that the elevator breaks. *[7 marks]*

(b) A factory makes life size wax copies of famous people. These famous people have weights that are independently, normally distributed with mean 80 kg and standard deviation 10 kg. The life size copies all have exactly the same weight as the famous person they represent. Twelve copies of one particular famous person are placed in the elevator in the factory. This elevator will also break if the total weight of the copies exceeds 1150 kg. Find the probability that the elevator breaks. *[7 marks]*

2. *[Maximum mark: 15]*

The length of time, T, in months, that a football manager stays in his job before he is removed can be approximately modelled by a normal distribution with population mean μ and population variance σ^2. An independent sample of five values of T is given below.

$$6.5, 12.4, 18.2, 3.7, 5.4$$

(a) Given that $\sigma^2 = 9$,

(i) use the above sample to find the 95 % confidence interval for μ, giving the bounds of the interval to two decimal places;

(ii) find the smallest number of values of T that would be required in a sample for the total width of the 90 % confidence interval for μ to be less than 2 months. *[10 marks]*

(b) If the value of σ^2 is unknown, use the above sample to find the 95 % confidence interval for μ, giving the bounds of the interval to two decimal places. *[5 marks]*

3. *[Maximum mark: 10]*

As soon as Sarah misses a total of 4 lessons at her school an email is sent to her parents. The probability that she misses any particular lesson is constant with a value of $\frac{1}{3}$. Her decision to attend a lesson is independent of her previous decisions.

(a) Find the probability that an email is sent to Sarah's parents after the 8^{th} lesson that Sarah was scheduled to attend. *[3 marks]*

(b) If an email is sent to Sarah's parents after the X^{th} lesson that she was scheduled to attend, find $\mathrm{E}(X)$. *[2 marks]*

(c) If after 6 of Sarah's scheduled lessons we are told that she has missed exactly 2 lessons, find the probability that an email is sent to her parents after a total of 12 scheduled lessons. *[3 marks]*

(d) If we know that an email was sent to Sarah's parents immediately after her 6^{th} scheduled lesson, find the probability that Sarah missed her 2^{nd} scheduled lesson. *[2 marks]*

4. *[Maximum mark: 9]*

A teacher has forgotten his computer password. He knows that it is either six of the letter J followed by two of the letter R (*i.e.* JJJJJJRR) or three of the letter J followed by four of the letter R (*i.e.* JJJRRRR). The computer is able to tell him at random just two of the letters in his password.

The teacher decides to use the following rule to attempt to find his password.

If the computer gives him a J and a J, he will accept the null hypothesis that his password is JJJJJJRR.

Otherwise he will accept the alternative hypothesis that his password is JJJRRRR.

(a) Define a Type I error. *[1 mark]*

(b) Find the probability that the teacher makes a Type I error. *[4 marks]*

(c) Define a Type II error. *[1 mark]*

(d) Find the probability that the teacher makes a Type II error. *[3 marks]*

5. *[Maximum mark: 12]*

The (partially completed) contingency table of observed values when doing a χ^2 test for independence is as follows, where $x \in \mathbb{N}$.

		Ability at rugby		
		Good	**Bad**	**Totals**
Ability at soccer	**Good**	x		75
	Bad			25
	Totals	60	40	100

(a) Copy and complete this table, filling in the missing values in terms of x. *[3 marks]*

(b) Complete a table for the expected values. *[2 marks]*

(c) Use the formula $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$ to find χ^2_{calc} in terms of x.

Give your answer in the form $\chi^2_{calc} = k(x-45)^2$ where k is a fraction that has to be determined. *[2 marks]*

(d) Let the null hypothesis be: "ability at soccer and ability at rugby are independent". If the null hypothesis is accepted at the 5 % significance level, find the possible values of x. *[5 marks]*